Avec le soutien de

Guaranteed estimation of the area swept by a sonar

Maël Godard, Luc Jaulin, Damien Massé

October 13, 2025

Introduction

Sonar (**SO**und **N**avigation **A**nd **R**anging) use **acoustic waves** to sense their environment.

Used in **maritime** and **underwater robotics** as it can be used in both clear and **turbid water**.

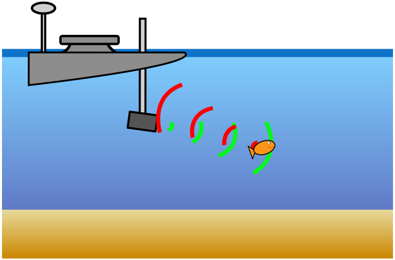

Figure 1: Sonar sensor

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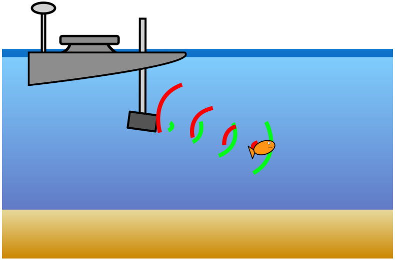

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Multibeam sonar

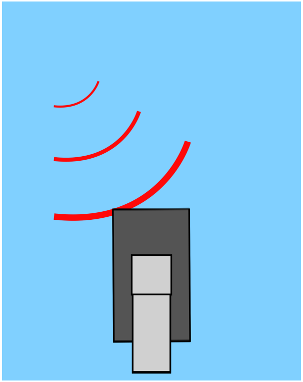

Figure 2: Wave reception

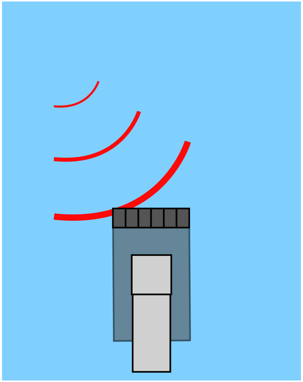

Figure 3: Multiple receptors

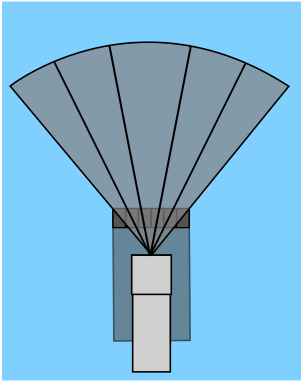

Figure 4: Beamforming

Fan view

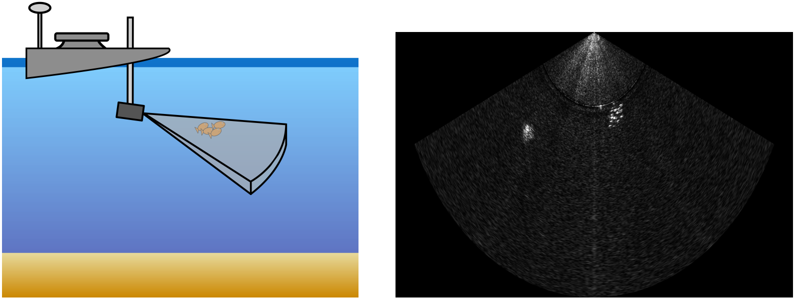

Figure 5: Fan view of a school of fish

Multibeam sonar in robotics

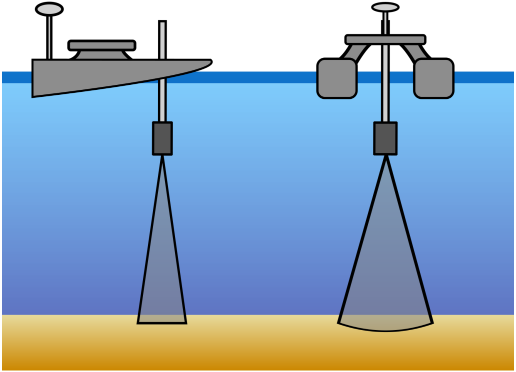

Figure 6: Scanning of the seabed

Method

Methodology

We know :

- The range of the sonar
- Its aperture angle
- The trajectory of the robot

We want :

- The volume insonified
- The area swept at a given depth

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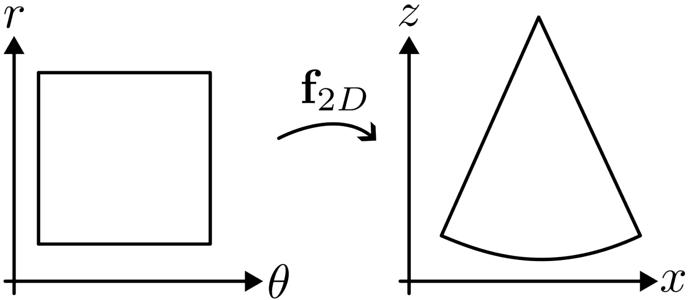

Figure 7: 2D geometry

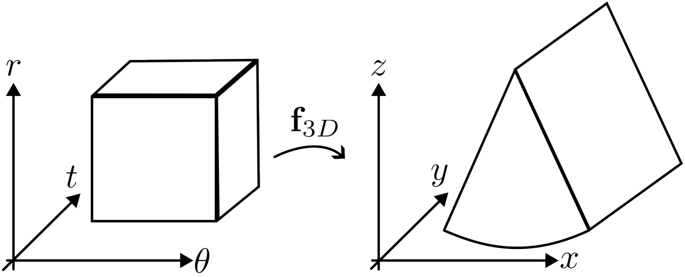

Figure 8: 3D geometry

Volume swept : PEIBOS

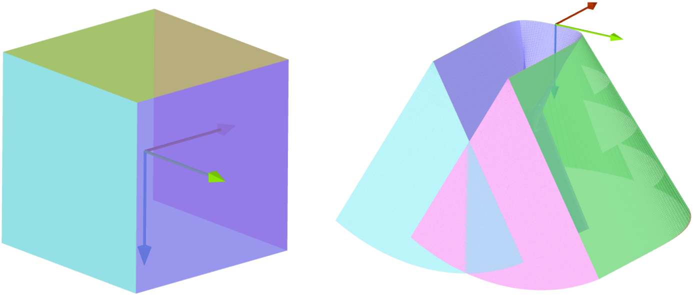

Figure 9: Volume swept for $r \in [0, 10]$, $\theta \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$, $t \in [0, 1]$ [1]

[1] Maël Godard, Luc Jaulin, Damien Massé, Inner and outer approximation of the image of a set by a nonlinear function, *International Journal of Approximate Reasoning*, Volume 187, 2025.

Projection at a given depth

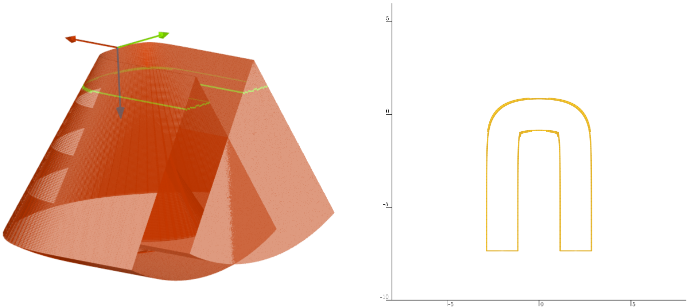

Figure 10: Projection at 1.5m

To CGAL [2]

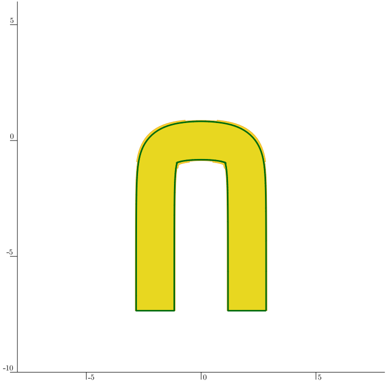

Figure 11: Polygon (yellow) with one hole (green)

[2] Susan Hert and Michael Seel. dD Convex Hulls and Delaunay Triangulations. In CGAL User and Reference Manual. CGAL Editorial Board, 6.0.1 edition, 2024.

Inner and outer approximation

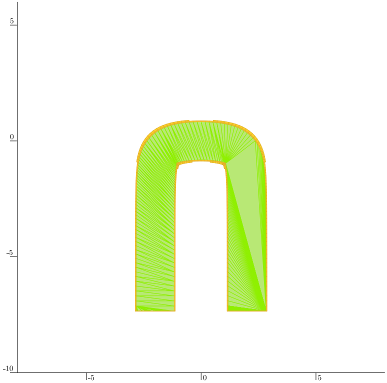

Figure 12: Inner (green) and outer (green+yellow) approximation of the area swept

Outside

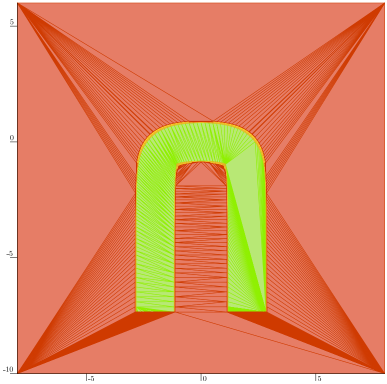

Figure 13: The red area is for sure not swept

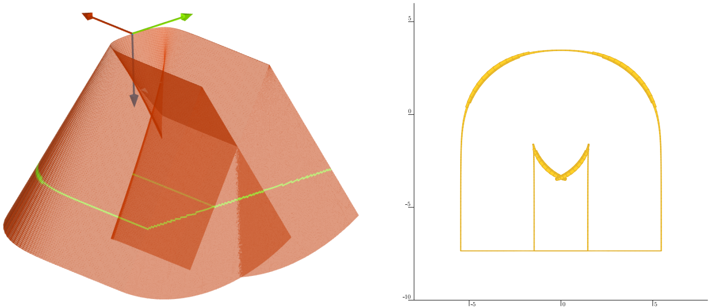

Figure 14: Projection at 6.0m

Multiple holes

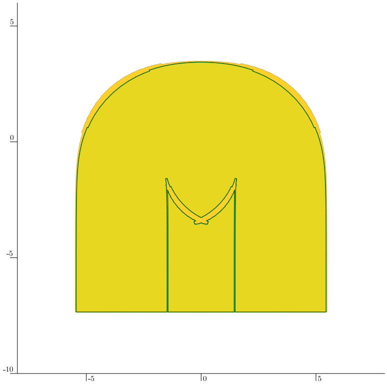

Figure 15: CGAL's polygon with hole

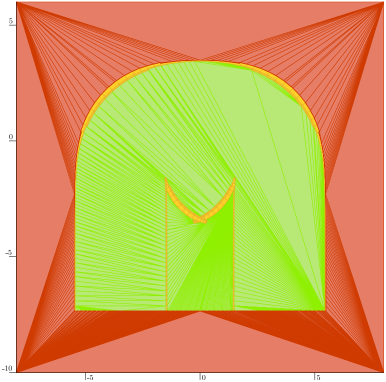

Figure 16: Fake boundaries

Removing fakes

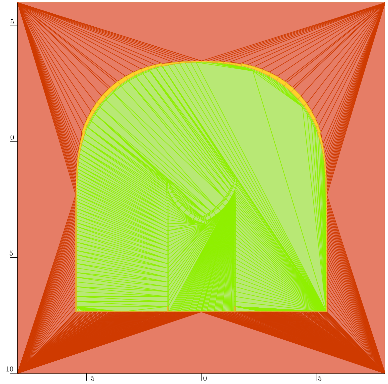

Figure 17: Fake boundaries removed

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- Computation of the volume swept by the sonar
- Projection and computation of the area swept
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Physical model

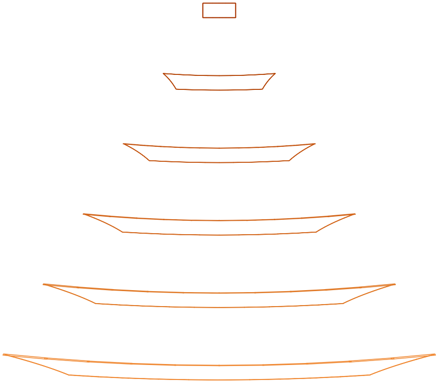

Figure 18: Propagation of an acoustic wave

Thank you for your attention


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