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Set-Based Identification of Characteristic Curves

Robex Seminar, Brest

DOI:10.13140/RG.2.2.22095.14247

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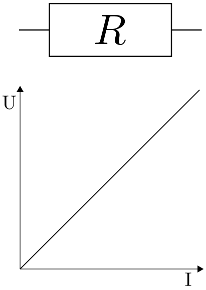
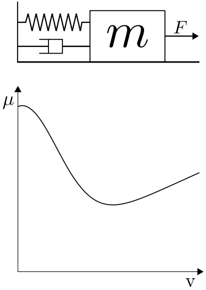
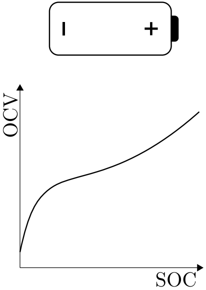
Objective







Objective

Research Question 1

Identification of characteristic curves . . .

- . . . without prior knowledge about the structure of the function representing this characteristic.
- . . . during system operation (offline and online identification).
- . . . without specifying excitation signals.

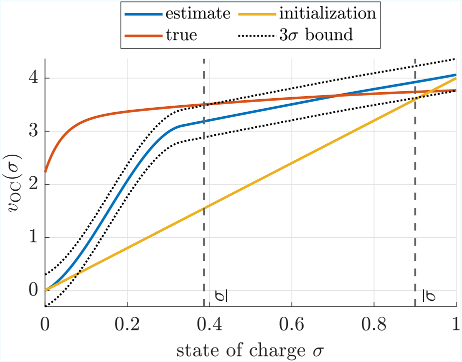
Research Question 2

Implementation of this identification process . . .

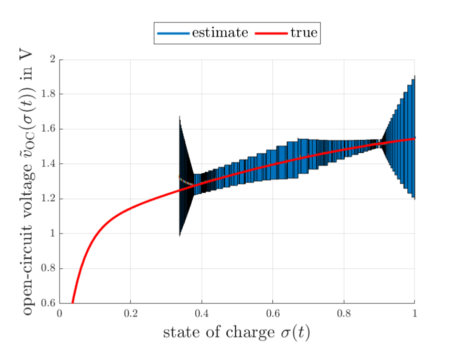
- . . . with a reasonable computational effort.
- . . . with reasonable memory requirements.
- . . . with providing guaranteed uncertainty bounds.

Objective

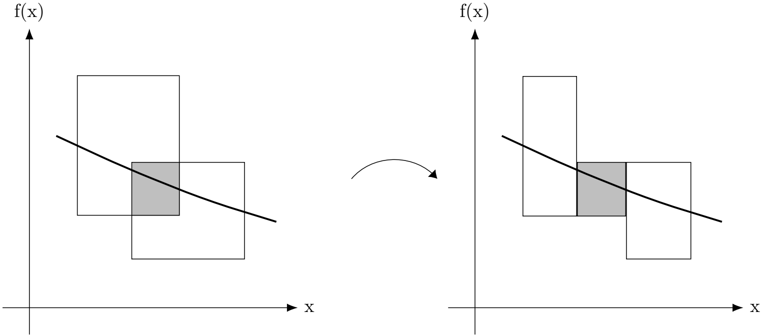
Stochastic



Set-based

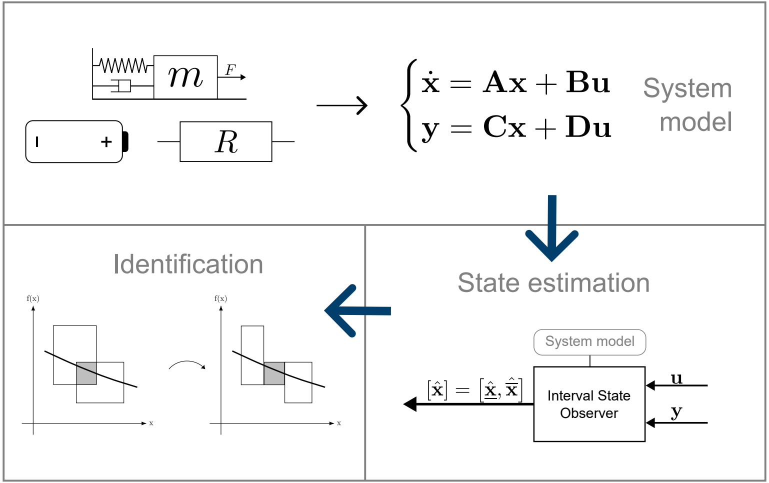


Identification

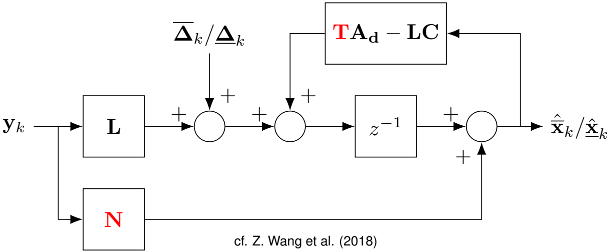


$$\mathcal{L} = \{([x_1], [y_1]), ([x_2], [y_2]), \ldots\}, \forall i, \forall x \in [x_i], \exists y \in [y_i], \text{ s.t. } y = f(x)$$

Identification



TNL Interval Observer



cf. Z. Wang et al. (2018)

$$\mathbf{T} + \mathbf{NC} = \mathbf{I}_{\mathrm{n}\times\mathrm{n}}$$

$$\Delta_k = g(\boldsymbol{w}_k, \boldsymbol{v}_k)\ ,\ \underline{\Delta}_k \leq \Delta_k \leq \overline{\Delta}_k$$

If $\mathbf{T} = \mathbf{I}_{\mathrm{n}\times\mathrm{n}}$ and $\mathbf{N} = \mathbf{0}$, the TNL observer has a Luenberger observer structure.
$\mathrm{w_k}$: process noise, $\mathrm{v_k}$: measurement uncertainty

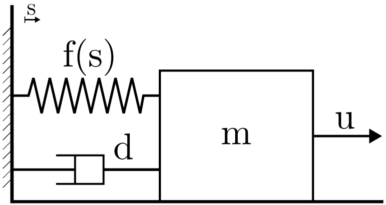
TNL Interval Observer

Linear matrix inequalities for the parameterization of the TNL observer	
stability criterion	based on a Lyapunov function
cooperativity criterion	observer system matrix has a Metzler structure
performance criterion	corresponds to the H_∞-like approach

M. LAHME, A. RAUH, AND G. DEFRESNE. Interval Observer Design for an Uncertain Time-Varying Quasi-Linear System Model of Lithium-Ion Batteries. Proc. of the 2024 European Control Conference (ECC), IEEE 3696-3702, 2024. DOI: 10.23919/ECC64448.2024.10591102

M. LAHME AND A. RAUH. Systematic Approach for Parameterizing TNL Interval Observers. 15th Summer Workshop on Interval Methods (SWIM 2024), Presentation, 2024. DOI: 10.13140/RG.2.2.33444.28800

Example: Nonlinear Spring Characteristic



$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1) - dx_2 + u \ , \quad f(x_1) = -x_1 - x_1^3$$

Example: Nonlinear Spring Characteristic

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1) - dx_2 + u \ , \ \ f(x_1) = -x_1 - x_1^3$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ [a_{21}] & -d \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

$$1 - x_1^2 \in [a_{21}]$$

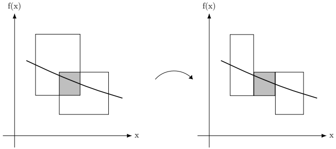
Example: Nonlinear Spring Characteristic

Interval Observer

Intersection

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ [a_{21}] & -d \end{bmatrix}$$

$$f(x_1) = [a_{21}]\, x_1$$



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Example: Nonlinear Spring Characteristic

Discretization

$$\mathbf{x}_k = \mathbf{A}_\mathrm{d} \cdot \mathbf{x}_{k-1} + \mathbf{B}_\mathrm{d} \cdot \mathrm{u}_{k-1}$$
$$\mathbf{y}_k = \mathbf{C} \cdot \mathbf{x}_k + \mathrm{v}_k$$

$$\mathbf{A}_\mathrm{d} = \mathbf{I}_\mathrm{n} + \mathrm{dt} \cdot \mathbf{A}\ , \ \mathbf{B}_\mathrm{d} = \mathrm{dt} \cdot \mathbf{B}$$

$$\mathbf{x}_k \in \mathbb{R}^\mathrm{n},\ \mathbf{y}_k \in \mathbb{R}^\mathrm{m},\ \mathrm{v}_k \in \mathbb{R}^\mathrm{m}$$

$$\underline{\mathrm{v}}_k \leq \mathrm{v}_k \leq \overline{\mathrm{v}}_k$$

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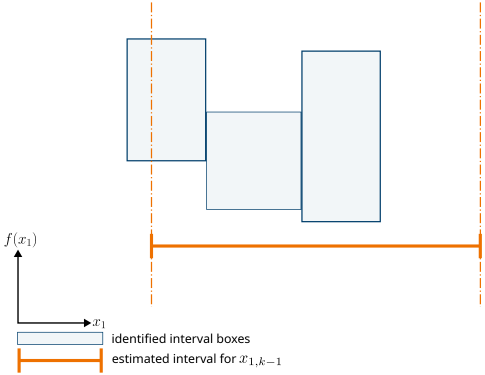
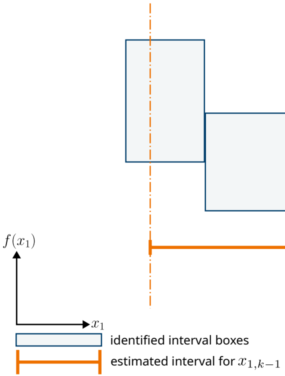


$$[\mathrm{a}_{21}]_{\mathrm{new}} = [\mathrm{a}_{21}]$$

Example: Nonlinear Spring Characteristic



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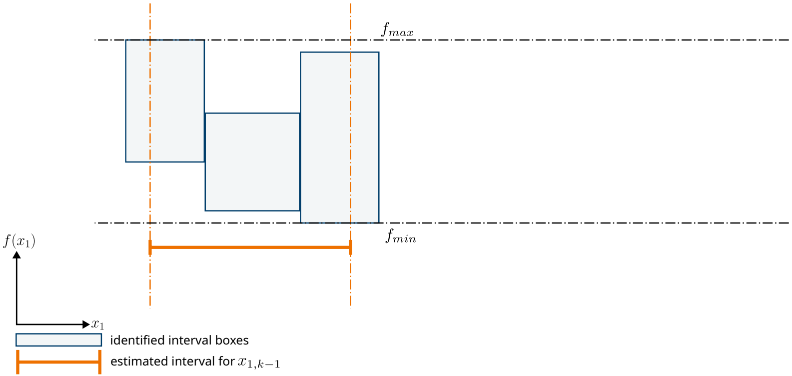

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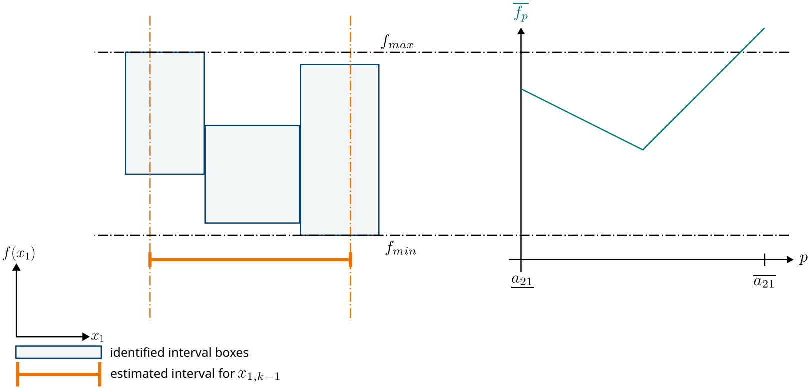

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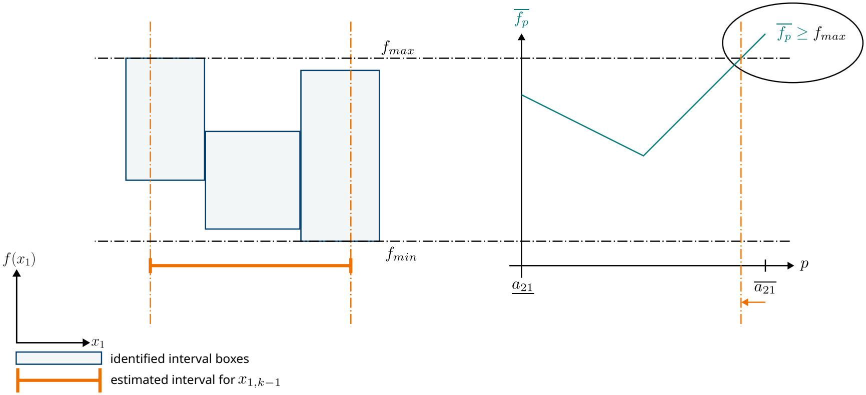
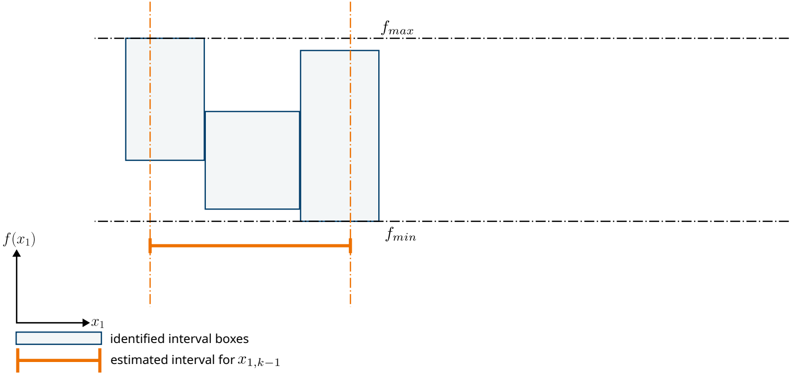
Example: Nonlinear Spring Characteristic



$$[a_{21}]_{new} = \left[\underline{a_{21}}\ ;\ \min\left\{p \in [a_{21}] \,\middle|\, \overline{f_p} \geq f_{max}\right\}\right]$$

Example: Nonlinear Spring Characteristic



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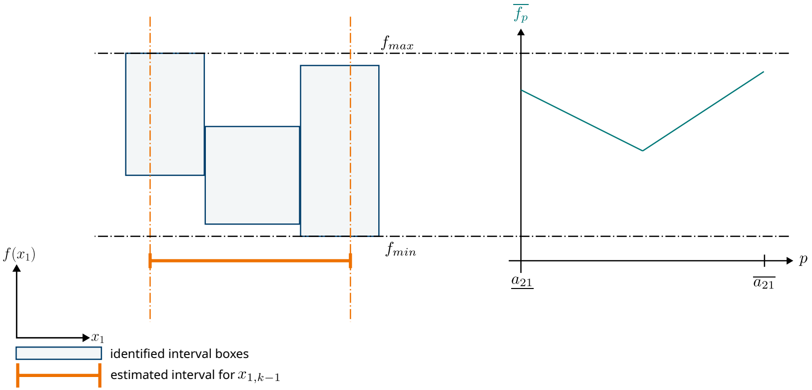

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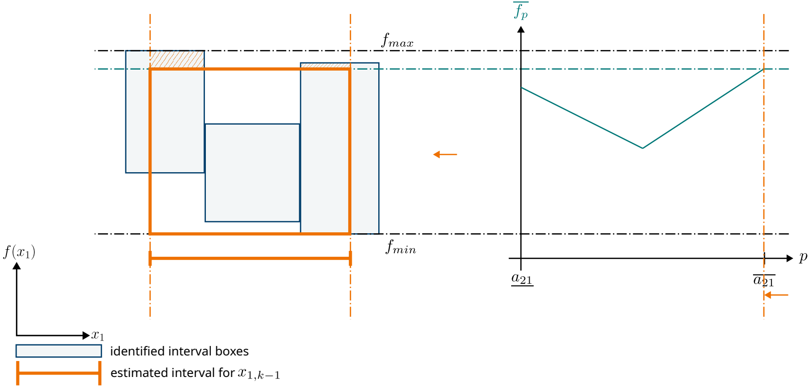
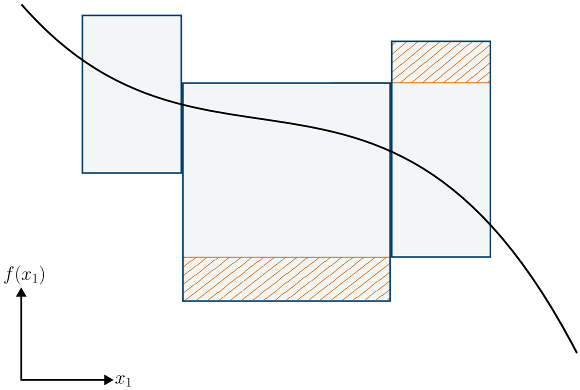
Example: Nonlinear Spring Characteristic



$$[a_{21}]_{new} = \left[\underline{a_{21}}\ ;\ \arg\max_{p \in [a_{21}]} \overline{f_p}\right]$$

Example: Nonlinear Spring Characteristic



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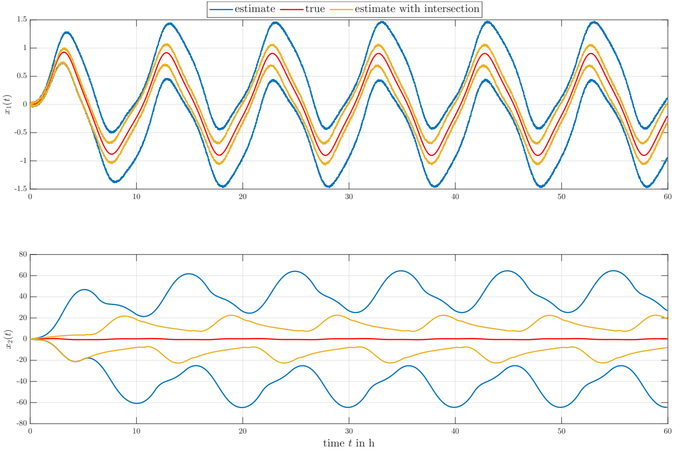

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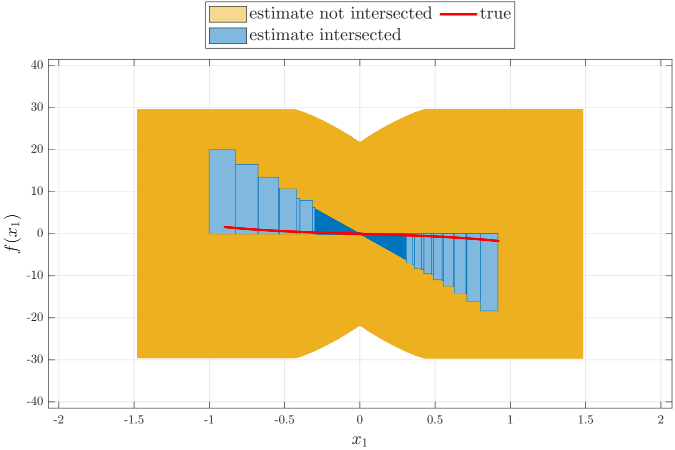
Example: Nonlinear Spring Characteristic



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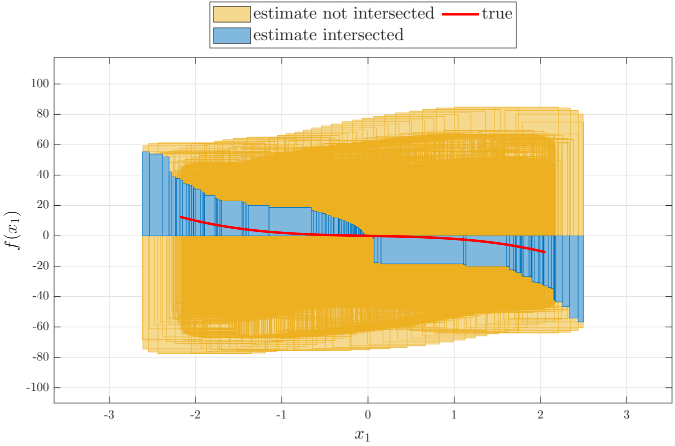
Example: Nonlinear Spring Characteristic

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \underbrace{f(x_1)}_{\color{red}{x_3}} - dx_2 + u \ , \quad f(x_1) = -x_1 - x_1^3 \\
\dot{x}_3 &= \frac{\partial f}{\partial x_1} x_2
\end{aligned}
$$

$$
\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -d & 1 \\ 0 & [a_{32}] & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \ , \quad y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}
$$

$$
\frac{\partial f}{\partial x_1} \in [a_{32}]
$$

Example: Nonlinear Spring Characteristic



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Conclusions and Future Work

- identification of a characteristic curve inside the state equation
- "Predictor-Corrector" like approach to improve the state estimation by estimating the uncertain interval in the system matrix $\mathbf{A}$

- enhance identification accuracy
- extension to the identification of a function that depends on two variables
- experimental validation

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Questions?

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www.uol.de/en/computingscience/dcis

Further Related Publications

[1] M. Lahme and A. Rauh. Set-Valued Approach for the Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries. *Acta Cybernetica (Special Issue of SWIM 2022)*, 26(4):855–869, 2024. DOI: 10.14232/actacyb.301185.

[2] M. Lahme and A. Rauh. Online Identification of the Open-Circuit Voltage Characteristic of Lithium-Ion Batteries with a Contractor-Based Procedure. *Proc. of the 27th International Conference on Methods and Models in Automation and Robotics (MMAR)*, IEEE 39–44, 2023. DOI: 10.1109/MMAR58394.2023.10242524.

[3] M. Lahme, A. Rauh, and G. Defresne. Interval Observer Design for an Uncertain Time-Varying Quasi-Linear System Model of Lithium-Ion Batteries. *Proc. of the 2024 European Control Conference (ECC)*, IEEE 3696-3702, 2024. DOI: 10.23919/ECC64448.2024.10591102.

[4] M. Lahme and A. Rauh. Combination of Stochastic State Estimation with Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries. *Proc. of the 1st IFAC Workshop on Control of Complex Systems (COSY)*, IFAC-PapersOnLine 55(40):97–102, 2022. DOI: 10.1016/j.ifacol.2023.01.055.

[5] M. Lahme and A. Rauh. Systematic Approach for Parameterizing TNL Interval Observers. *15th Summer Workshop on Interval Methods (SWIM 2024)*, Presentation, 2024. DOI: 10.13140/RG.2.2.33444.28800.

[6] T. Raïssi and D. Efimov. Some Recent Results on the Design and Implementation of Interval Observers for Uncertain Systems. *at - Automatisierungstechnik* (vol. 66) 3, 213–224, 2018. DOI: 10.1515/auto-2017-0081.

[7] Z. Wang, C.-C. Lim, and Y. Shen. Interval Observer Design for Uncertain Discrete-Time Linear Systems. *Systems & Control Letters*, 116:41–46, 2018. DOI: 10.1016/j.sysconle.2018.04.003