Control of an enemy swarm by broadcasting false messages

L. Jaulin

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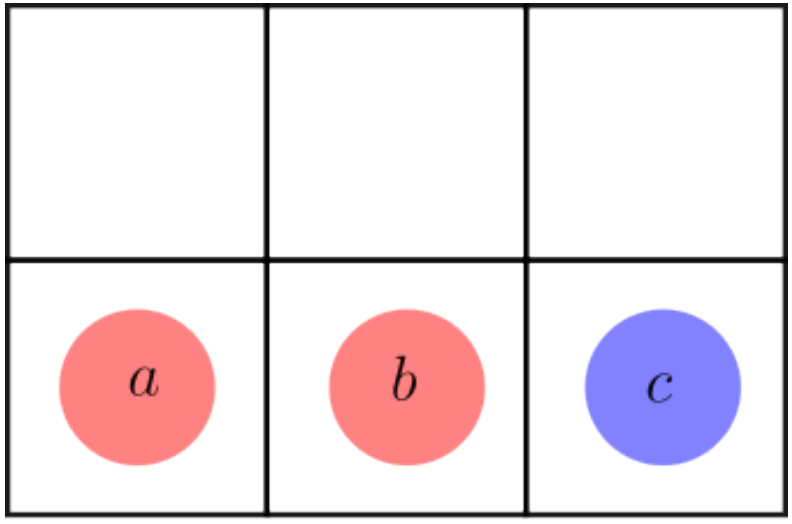
1. Dynamic Epistemic Logic

Bernardo Hummes Flores,
Méthodes formelles pour la robotique mobile.
Supervised by Eric Goubault, Sylvie Putot and Luc Jaulin

Muddy Children Problem

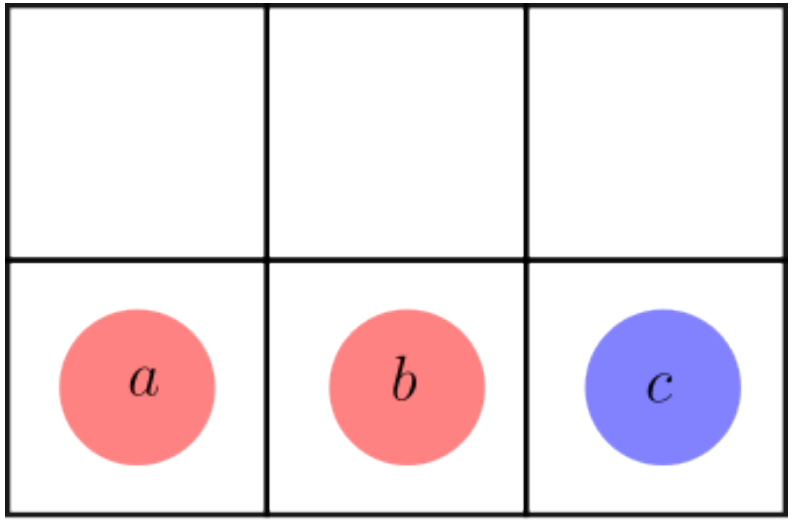


$$k = 0$$

Three robots a, b, c, blue and red can see each other.

They do not know their own color.

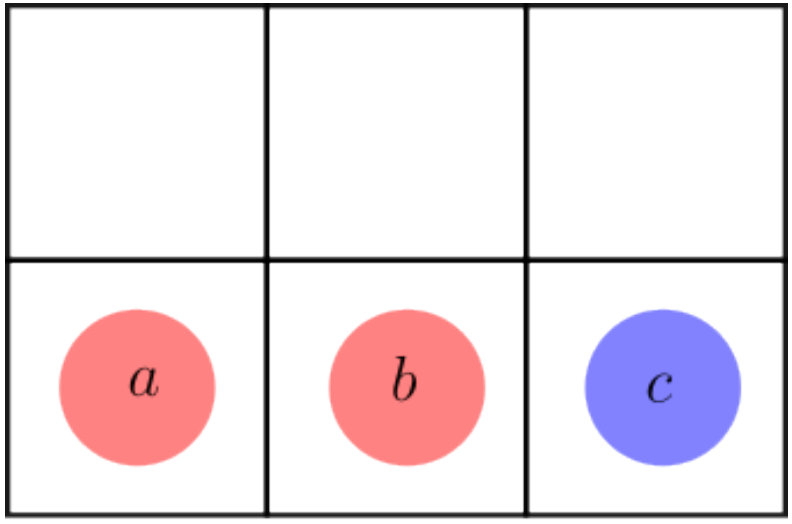
They cannot communicate



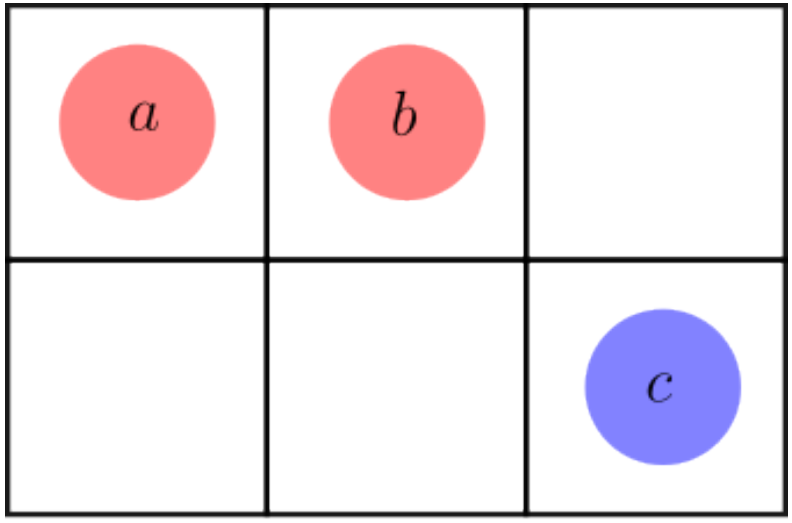
$k = 0$

God says: At least one of you is red.

If you know you are red : go up

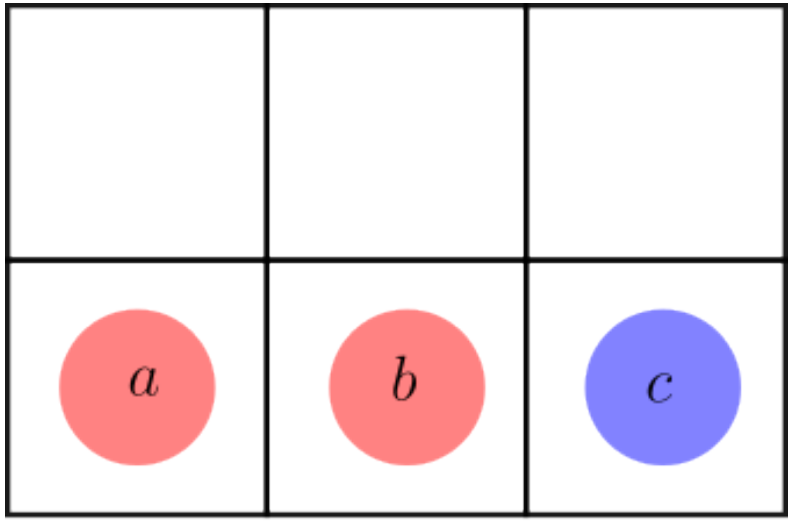


$$k = 1$$



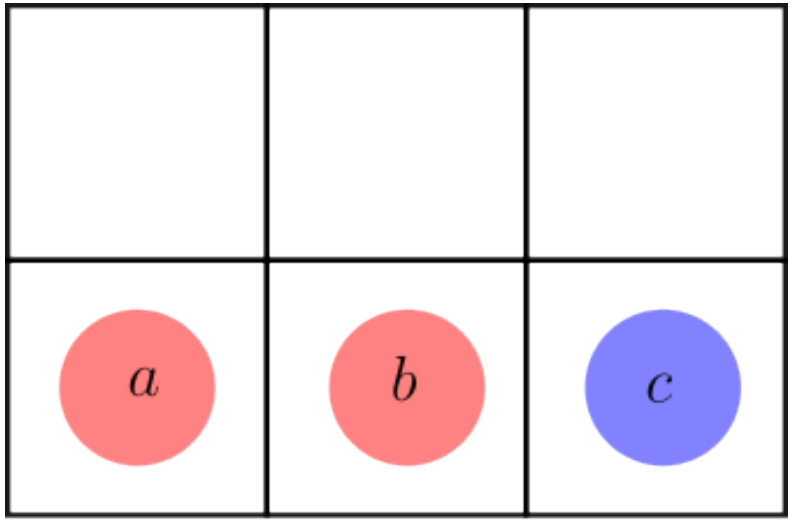
$$k = 2$$

Epistemic logic



$$k = 0$$

$$(i) \qquad K_a K_b \text{color}(c) = \text{blue}$$
$$(ii) \qquad K_c K_b \text{color}(c)$$

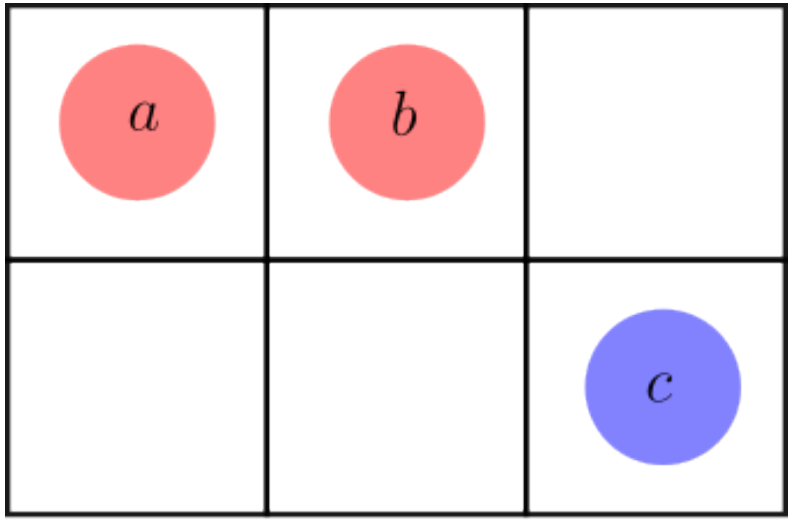


$$k = 1$$

(i) $K_b \text{color}(b) = \text{red}$

(ii) $\neg K_c K_b \text{color}(b) = \text{red}$

(iii) $\neg K_c \text{color}(c) = \text{blue}$



$$k = 2$$

$$(i) \qquad E\text{color}(a) = \text{red}$$
$$(ii) \qquad E\text{color}(c) = \text{blue}$$

$$K_1 K_2 \phi \Rightarrow K_1 \phi$$

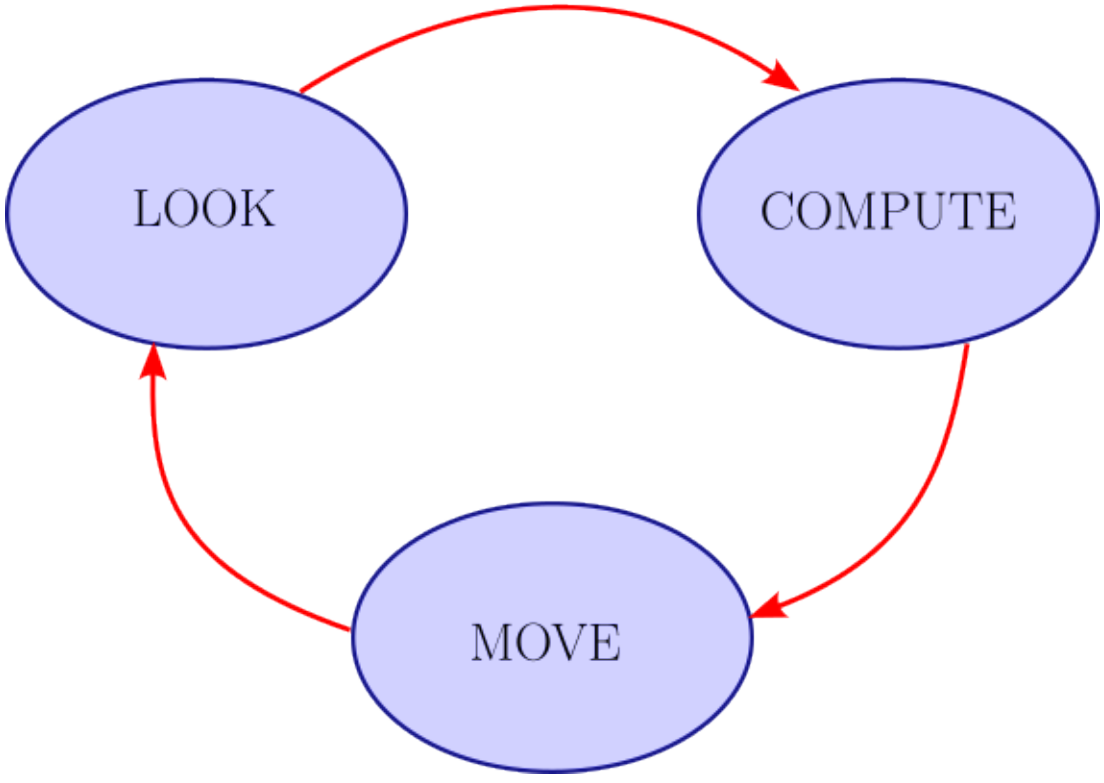
General knowledge

$$E\phi \Leftrightarrow \bigwedge_i K_i\phi$$

Common knowledge

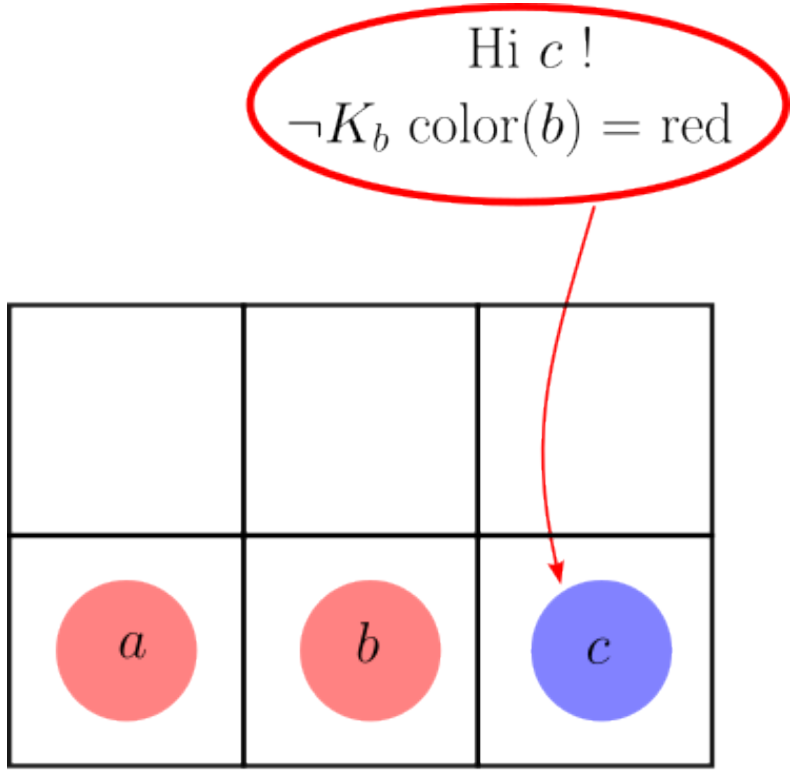
$$C\phi \Leftrightarrow \bigwedge_k E^k \phi$$

Look-Compute-Move model


LOOK
COMPUTE
MOVE

Control

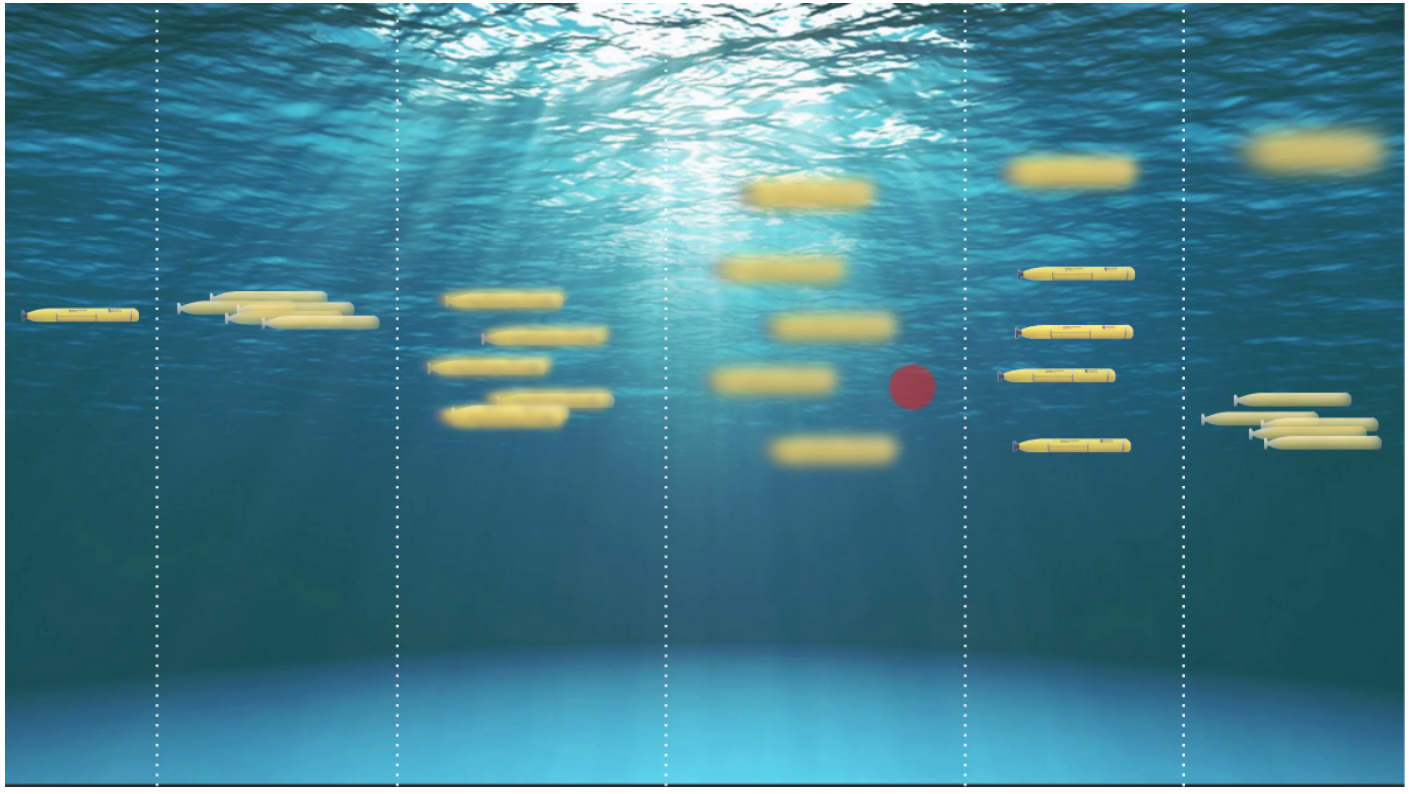
Send false or true messages to control the swarm



$k = 1$

- The colors of the robots is the ontic state
- They memory is the epistemic state

2. Ontic and epistemic states

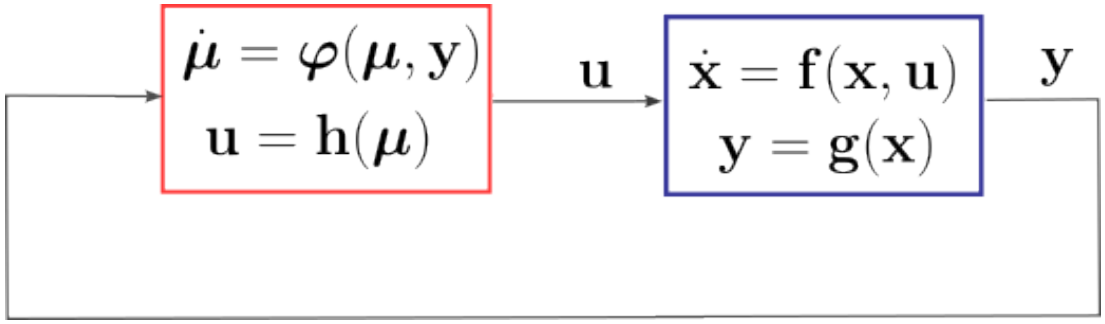


A robot with memory:

$$
\begin{array}{lcll}
\dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) & \text{(ontic evolution)} \\
\mathbf{y}(t) & = & \mathbf{g}(\mathbf{x}(t)) & \text{(observation)} \\
\mu_{k+1} & = & \varphi(\mu_k, \mathbf{y}(t_k)) & \text{(epistemic evolution)} \\
\mathbf{u}(t_k) & = & \mathbf{h}(\mu_k) & \text{(control)}
\end{array}
$$

With an analog controller

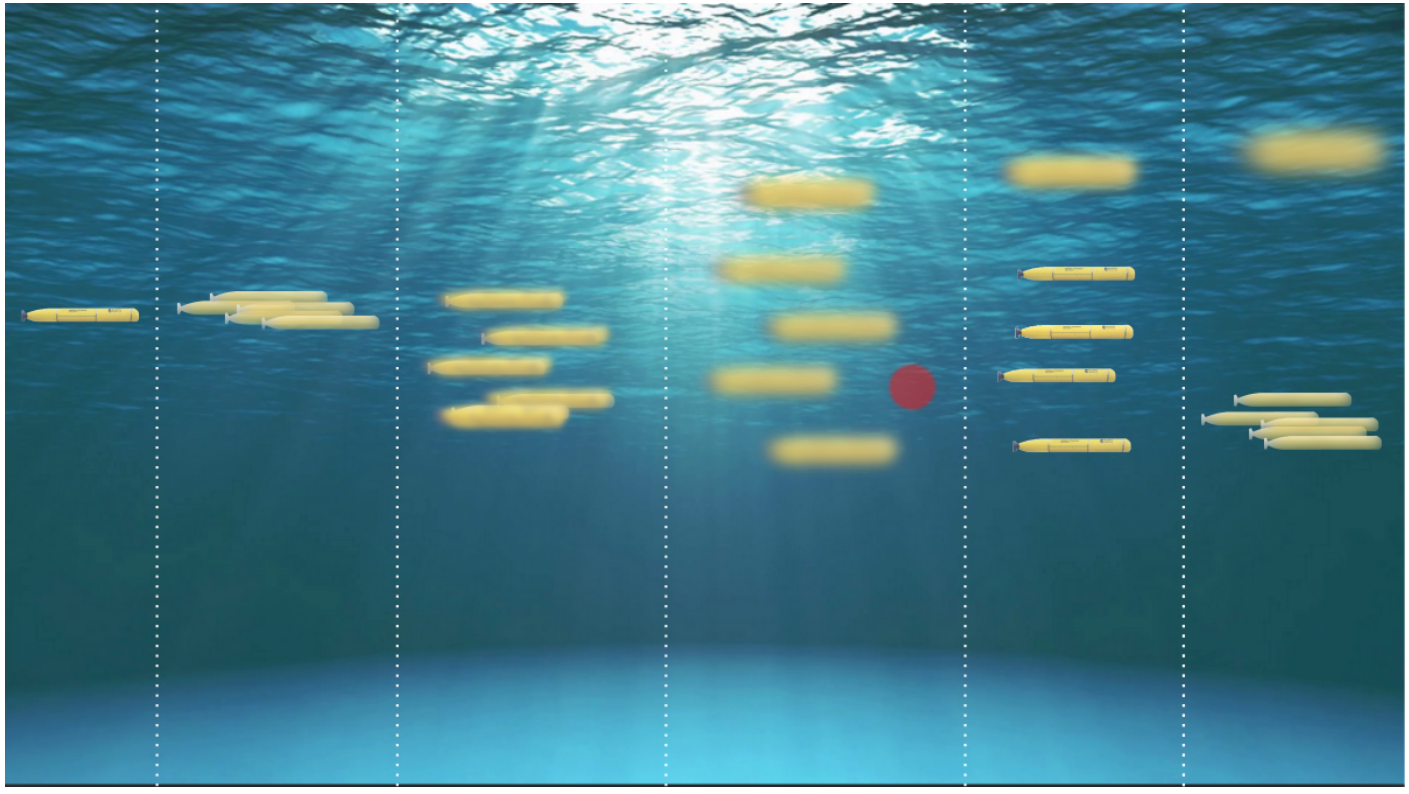
$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) && \text{(ontic evolution)} \\
\mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) && \text{(observation)} \\
\dot{\mu}(t) &= \varphi(\mu(t), \mathbf{y}(t)) && \text{(epistemic evolution)} \\
\mathbf{u}(t) &= \mathbf{h}(\mu(t)). && \text{(control)}
\end{aligned}
$$



$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{h}(\mu(t))) \quad \text{(ontic evolution)}$$
$$\dot{\mu}(t) = \varphi(\mu(t), \mathbf{g}(\mathbf{x}(t))) \quad \text{(epistemic evolution)}$$

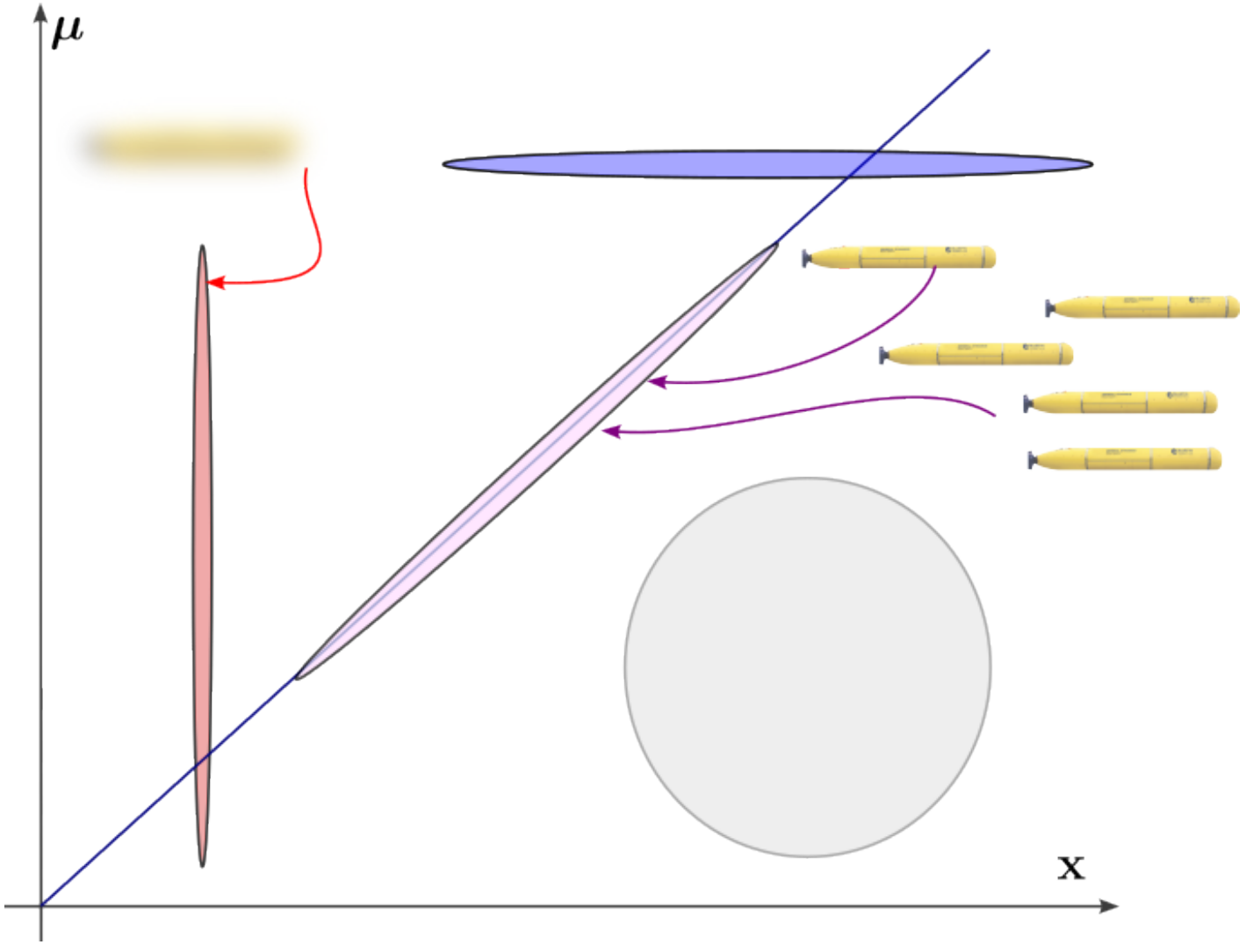
The global state is $\mathbf{z} = (\mathbf{x}, \mu)$. We have

$$\dot{\mathbf{z}} = \psi(\mathbf{z})$$



State of the robot : $\mathbf{z} = (\mathbf{x}, \mu)$, with

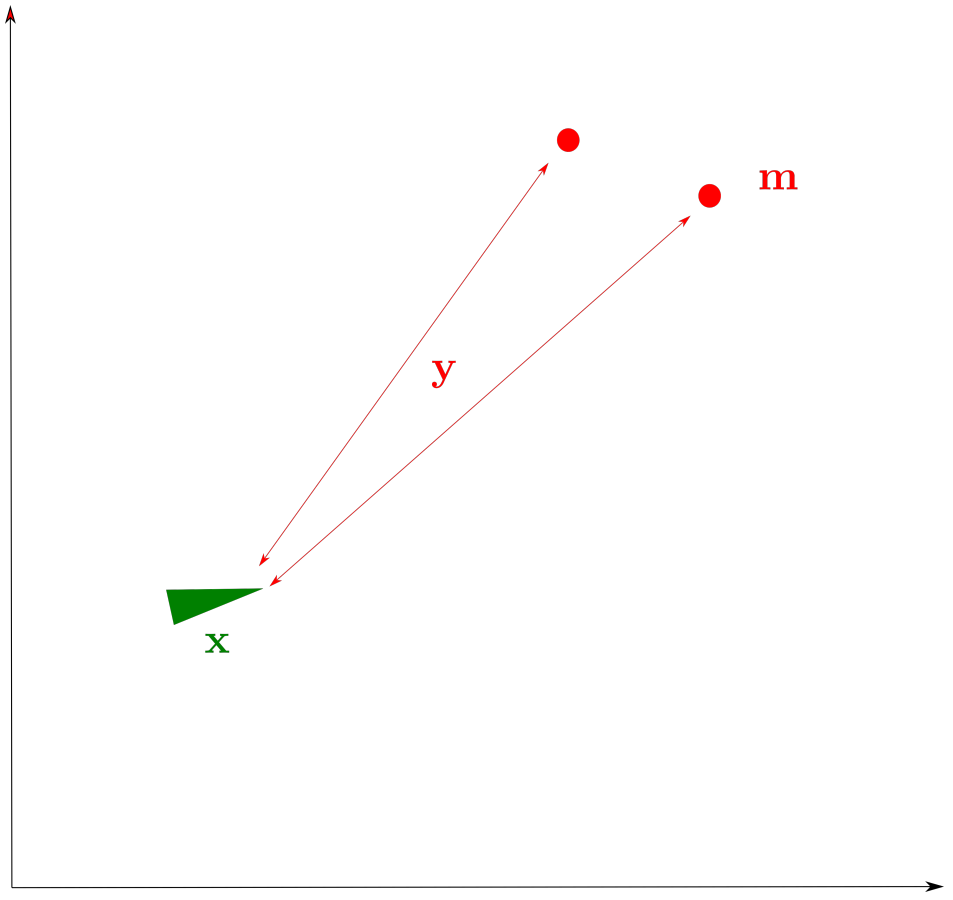
- $\mathbf{x}$: the ontic state
- μ: the epistemic state


μ
x

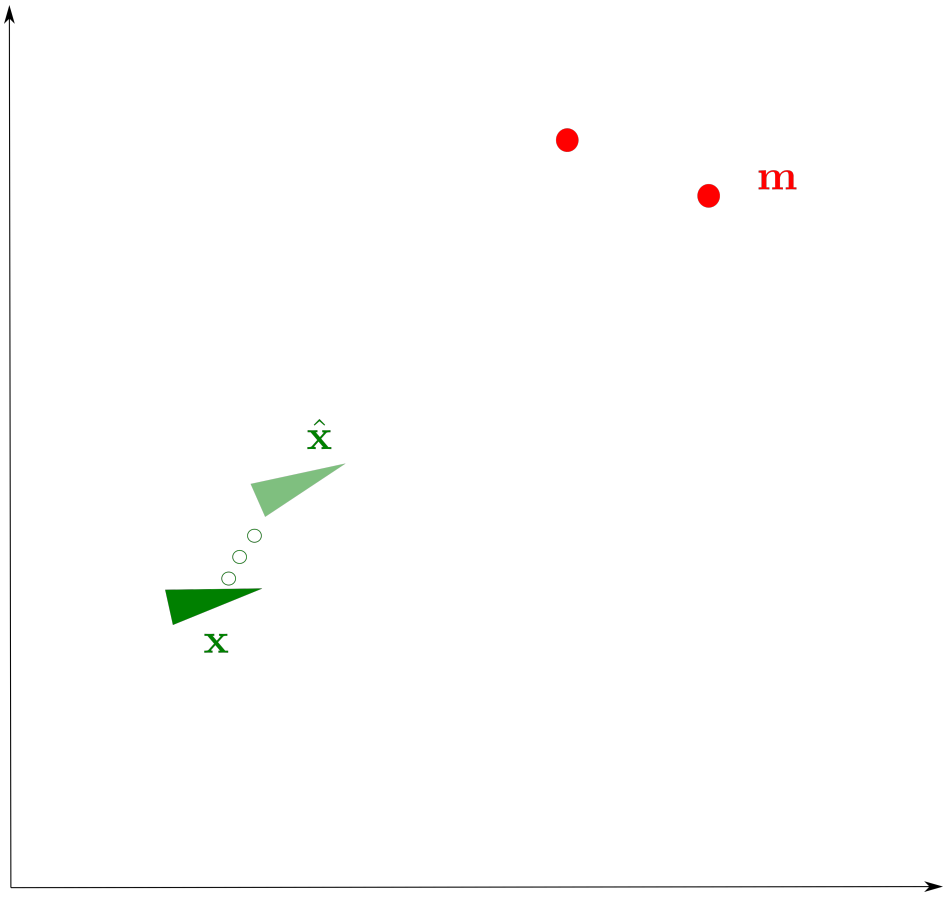
Perception : We measure $\mathbf{x}$

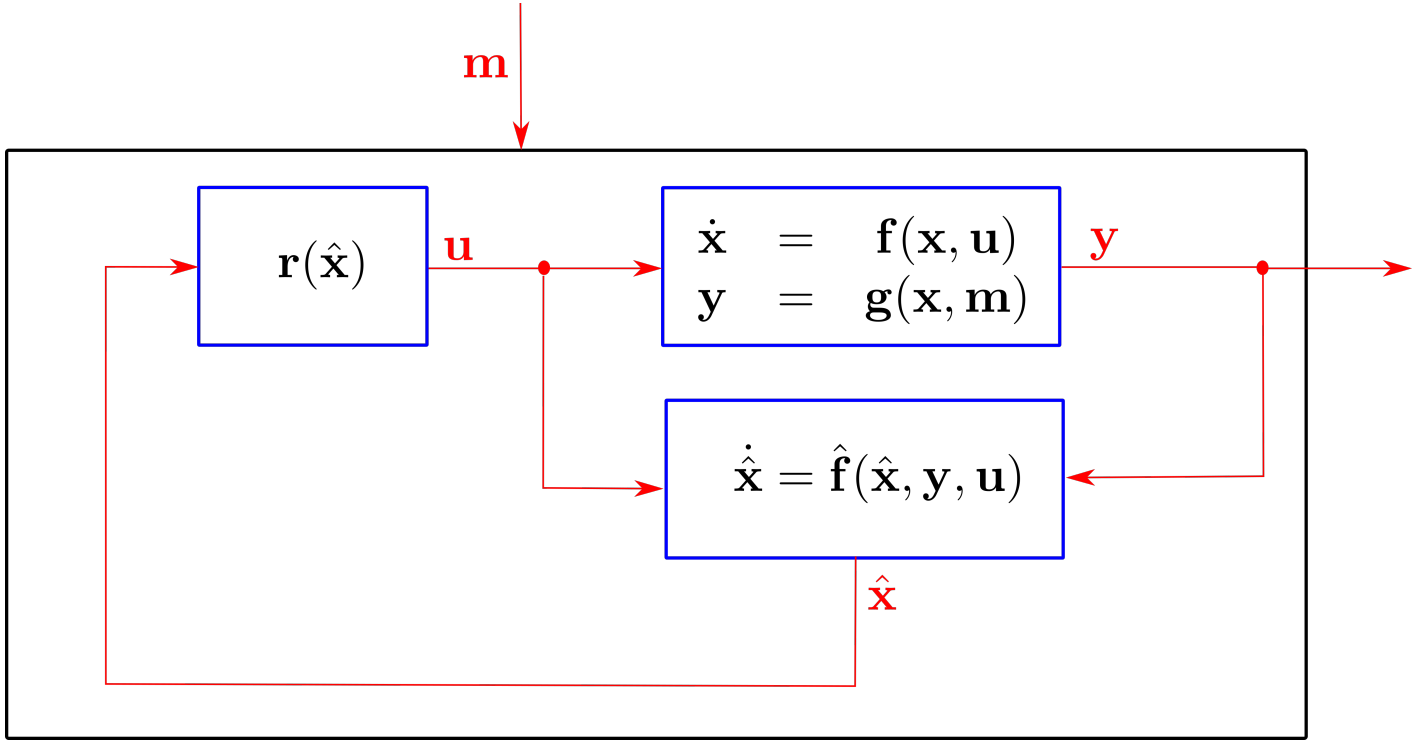
Communication : we measure μ

3. Attack


m
y
x

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{g}(\mathbf{x}) \end{aligned}$$


m
x̂
x


m
r(x̂)
u
ẋ = f(x, u)
y = g(x, m)
y
x̂˙ = f̂(x̂, y, u)
x̂

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
\mathbf{y} &= \mathbf{g}(\mathbf{x}, \mathbf{m}) \\
\dot{\hat{\mathbf{x}}} &= \hat{\mathbf{f}}(\hat{\mathbf{x}}, \mathbf{y}, \mathbf{u}) \\
\mathbf{u} &= \mathbf{r}(\hat{\mathbf{x}})
\end{aligned}
$$

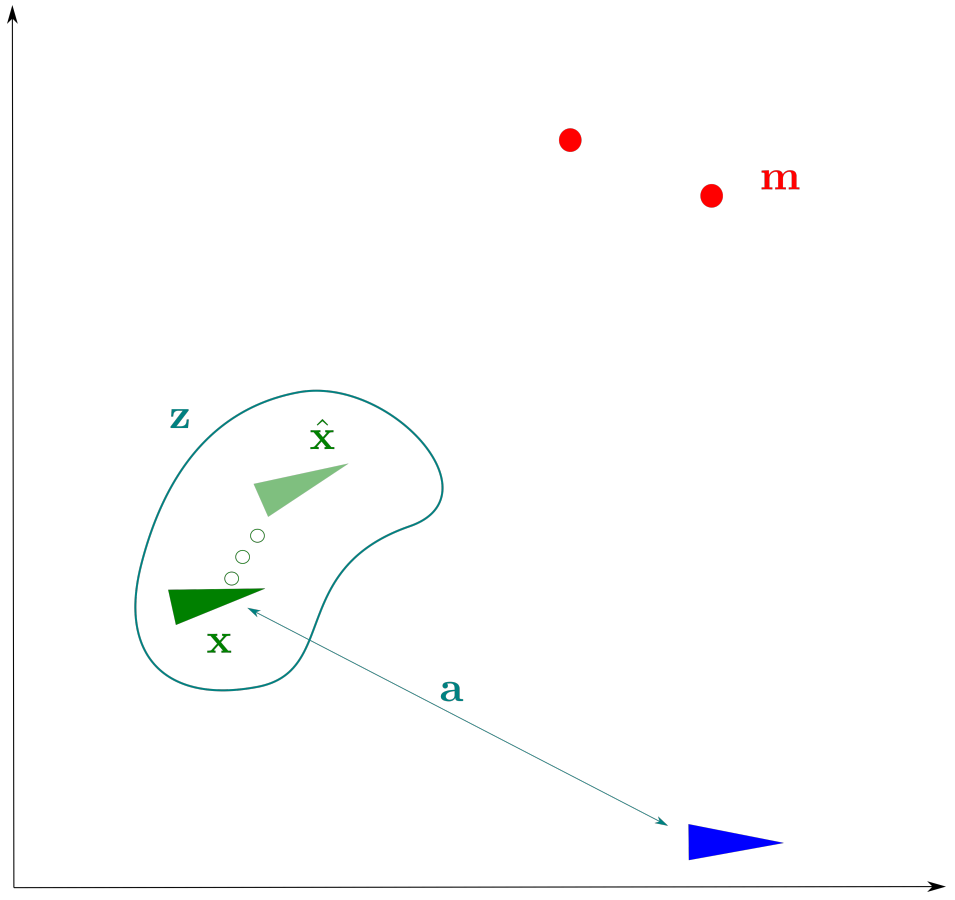
If we set $\mathbf{z} = (\mathbf{x}, \hat{\mathbf{x}})$, we get

$$\dot{\mathbf{z}} = \psi(\mathbf{z}, \mathbf{m})$$

Assume that we can observe the motion of the robot

$$\begin{aligned} \dot{\mathbf{z}} &= \psi(\mathbf{z}, \mathbf{m}) \\ \mathbf{a} &= \eta(\mathbf{x}) \end{aligned}$$

where $\mathbf{z} = (\mathbf{x}, \hat{\mathbf{x}})$.

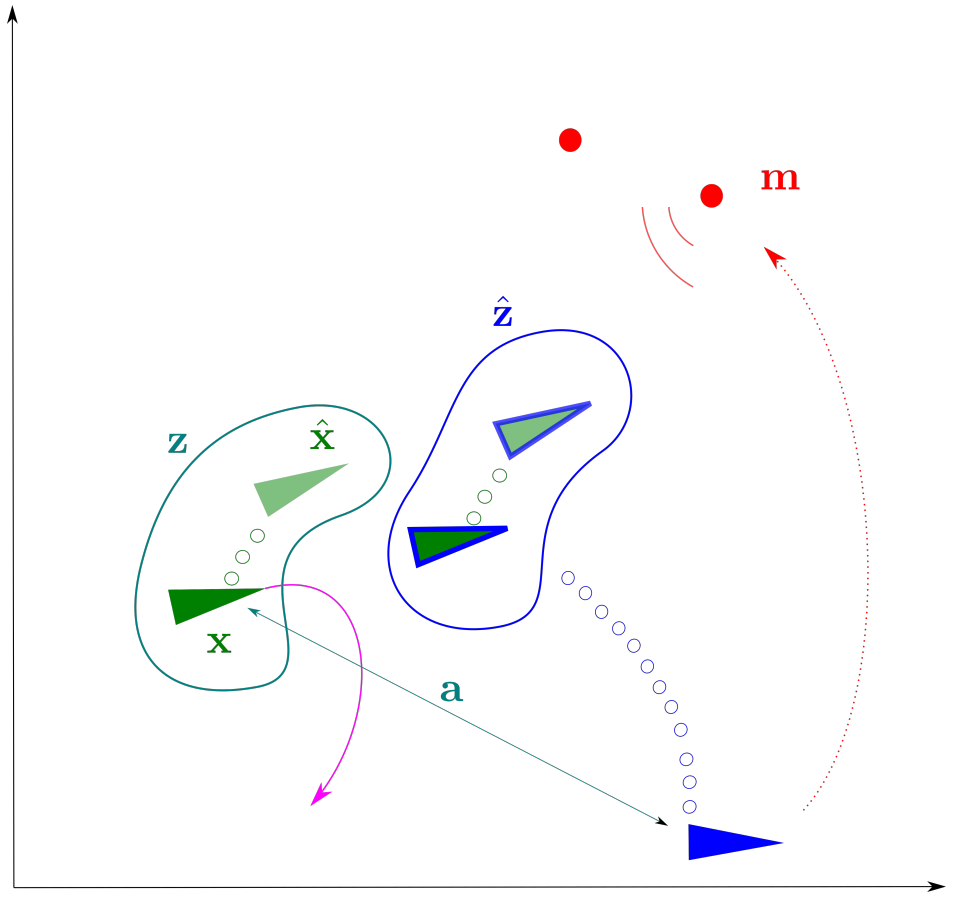

m
z
x̂
x
a

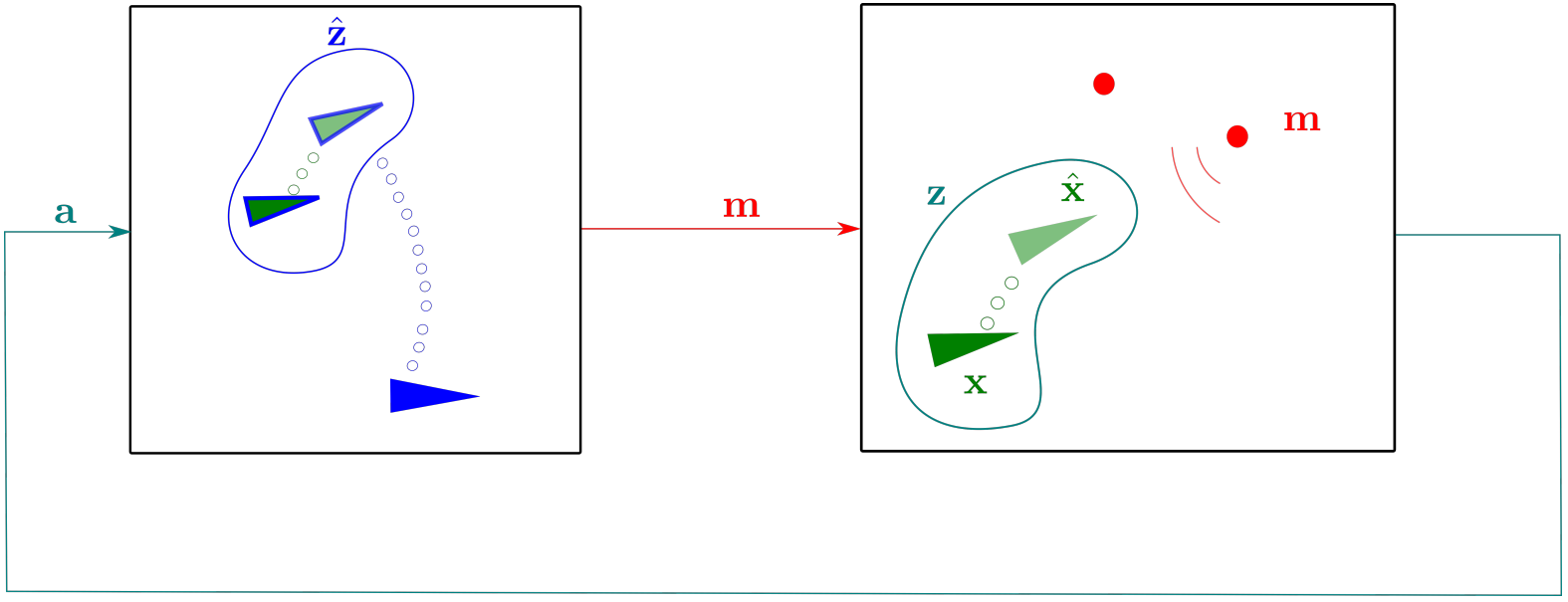
We can build an observer for $\mathbf{z}$:

$$\begin{aligned} \dot{\mathbf{z}} &= \psi(\mathbf{z}, \mathbf{m}) \\ \mathbf{a} &= \eta(\mathbf{x}) \\ \dot{\hat{\mathbf{z}}} &= \hat{\psi}(\mathbf{a}, \hat{\mathbf{z}}) \end{aligned}$$

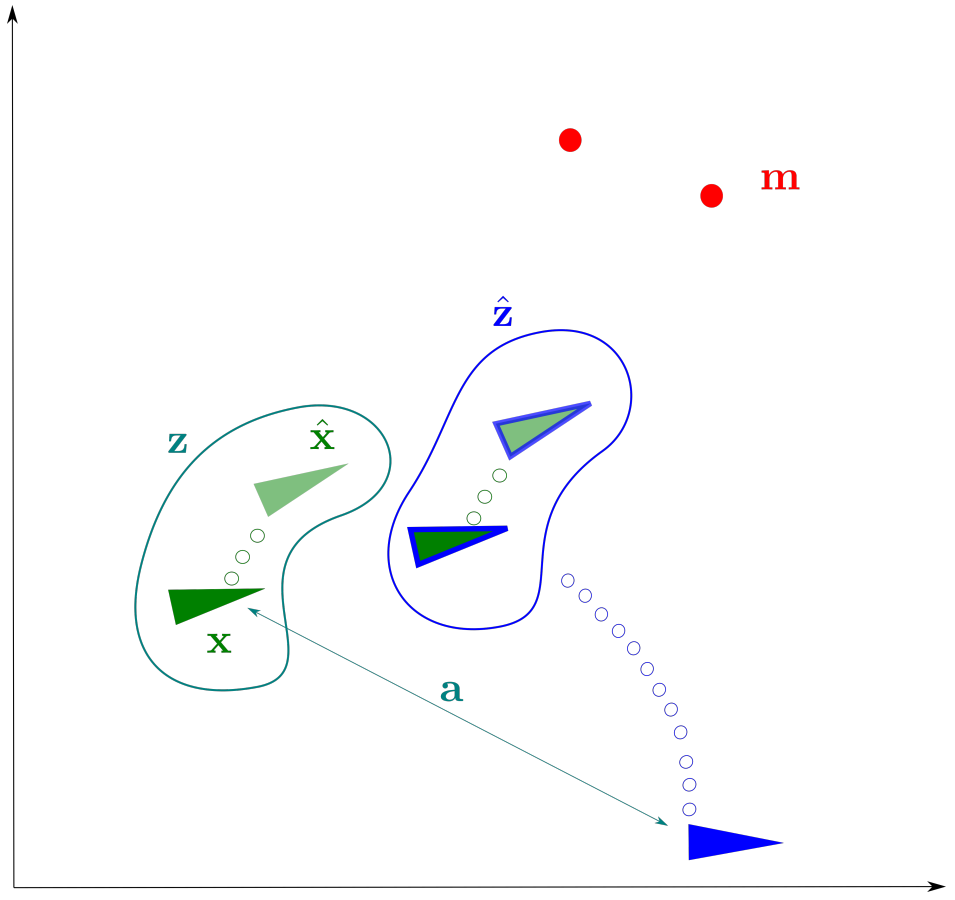
Use $\mathbf{m}$ as an input to control $\mathbf{x}$.
The vector $\mathbf{m}$ corresponds to the position of the landmark.
We want to build a controller:

$$\mathbf{m} = \rho(\hat{\mathbf{z}}, t)$$

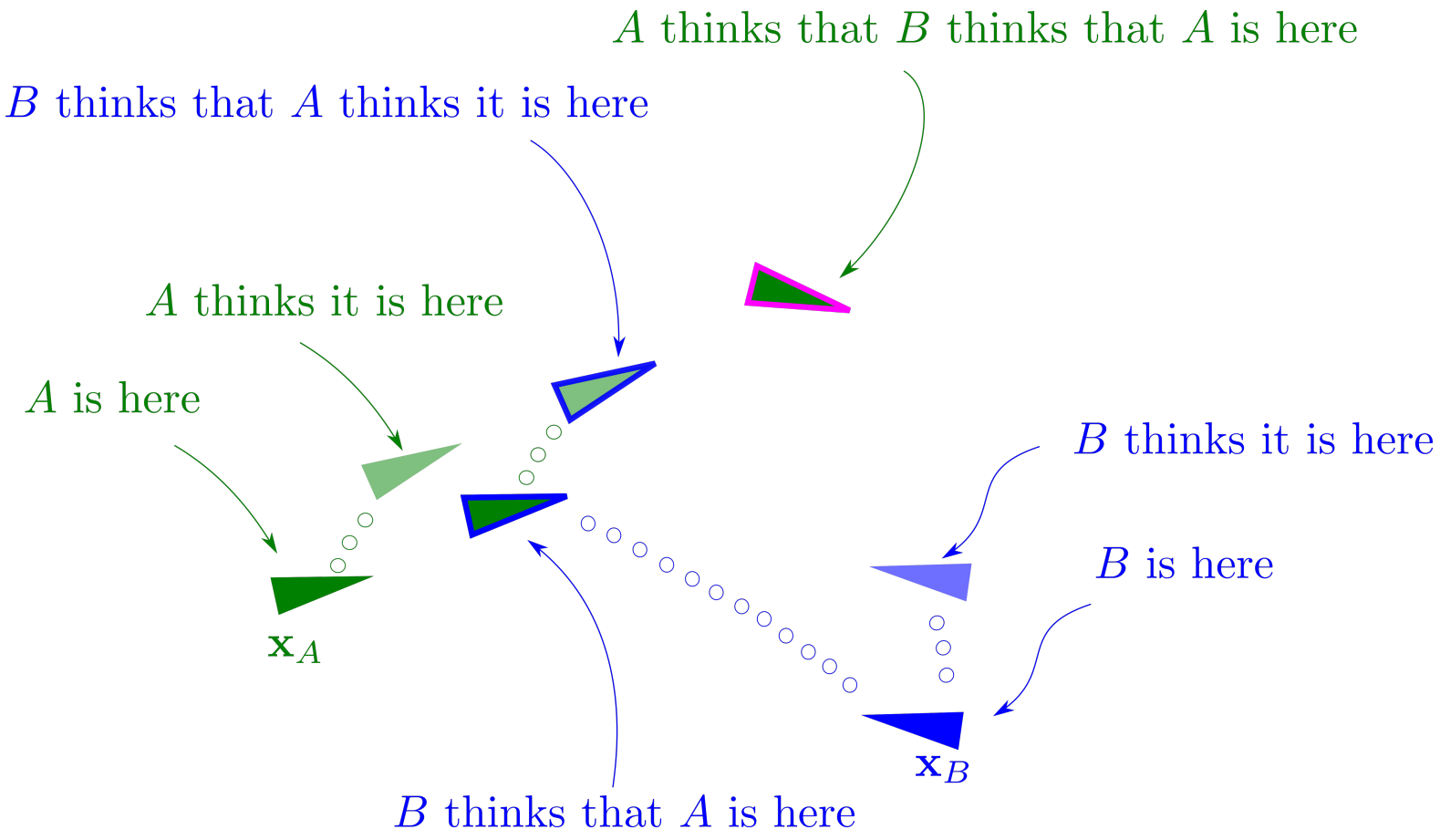

m
ẑ
z
x̂
x
a


ẑ
a
m
m
z
x̂
x

3. Swarm

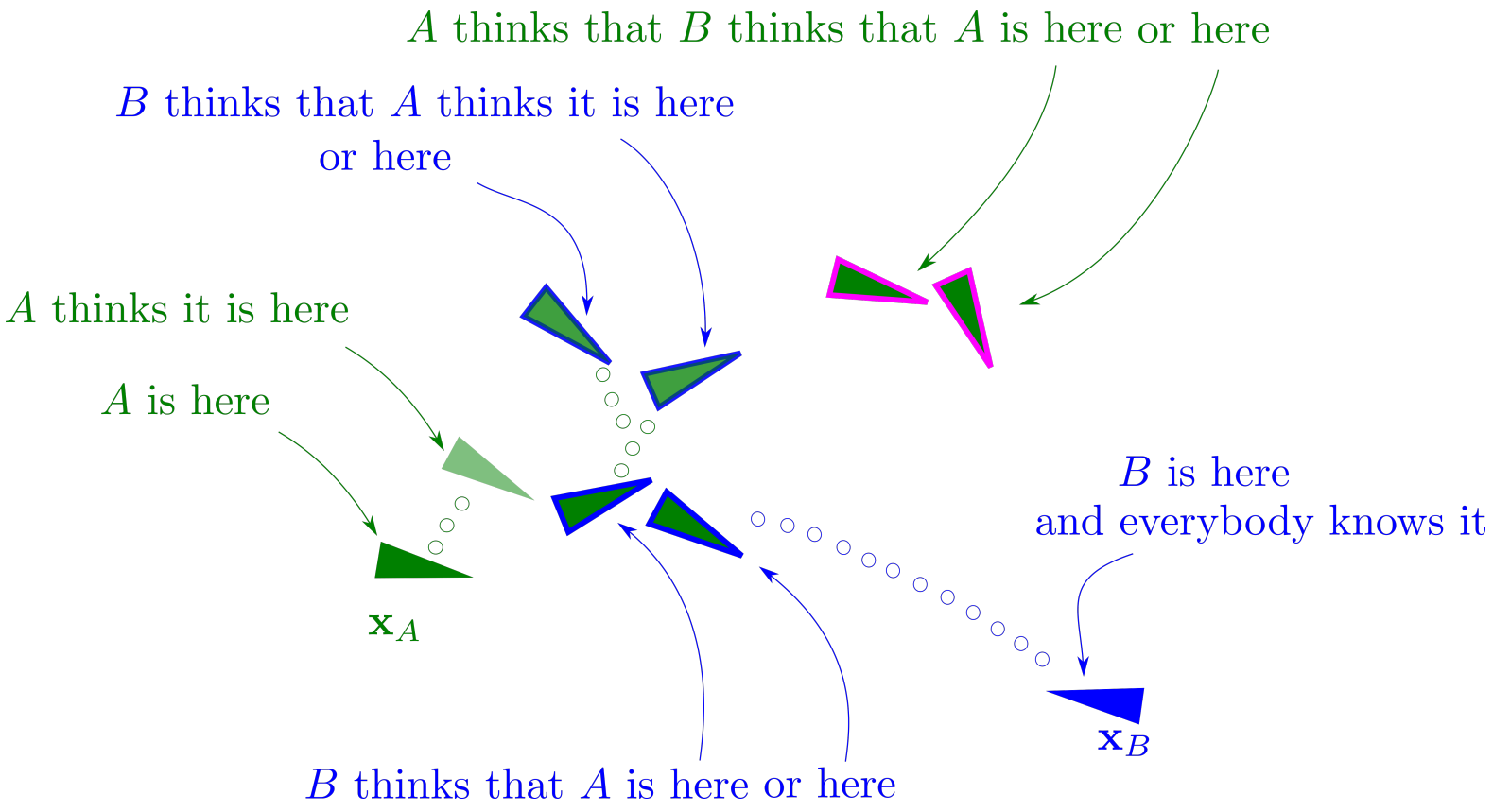

m
ẑ
z
x̂
x
a

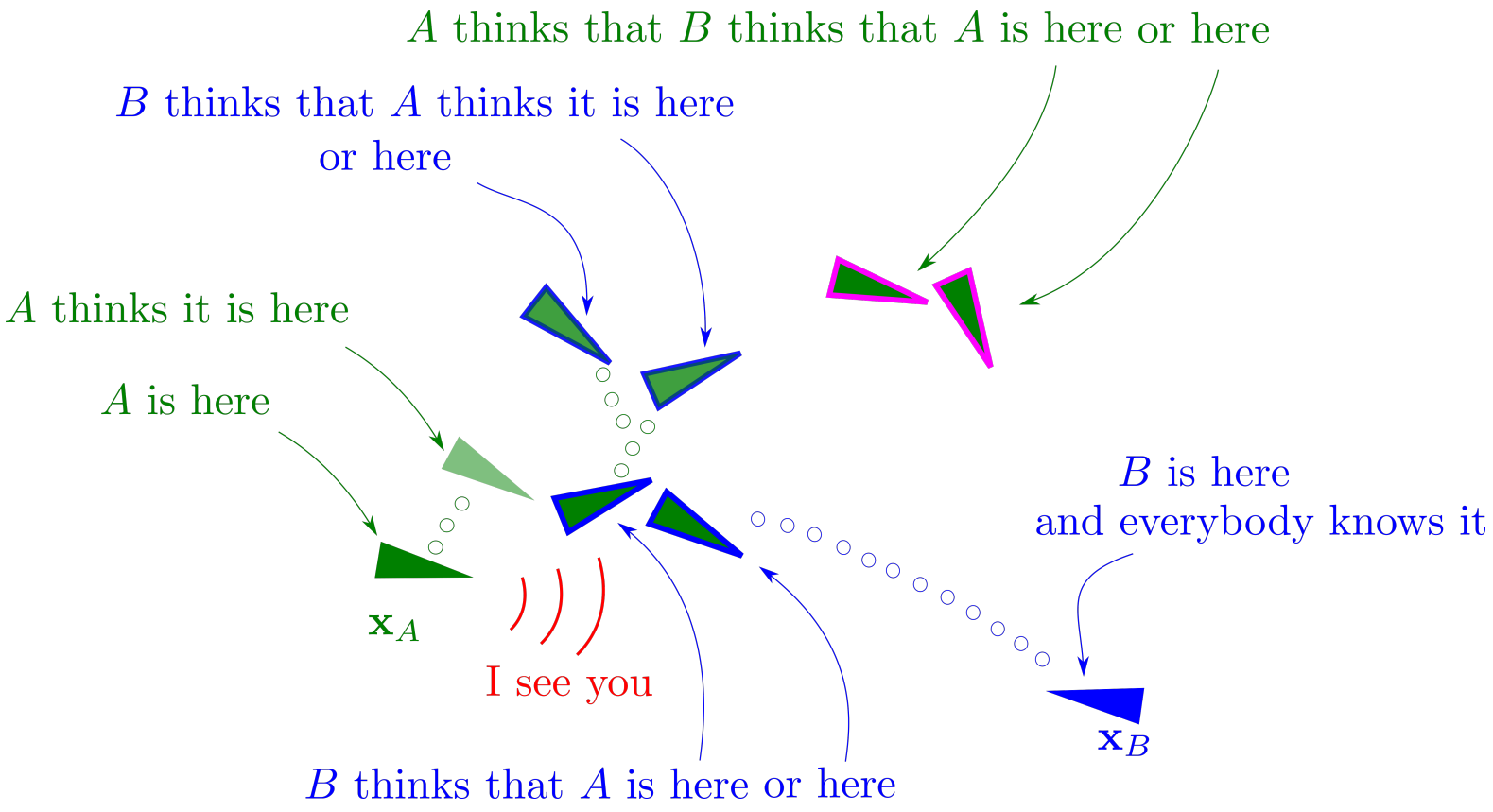
Distributed knowledge

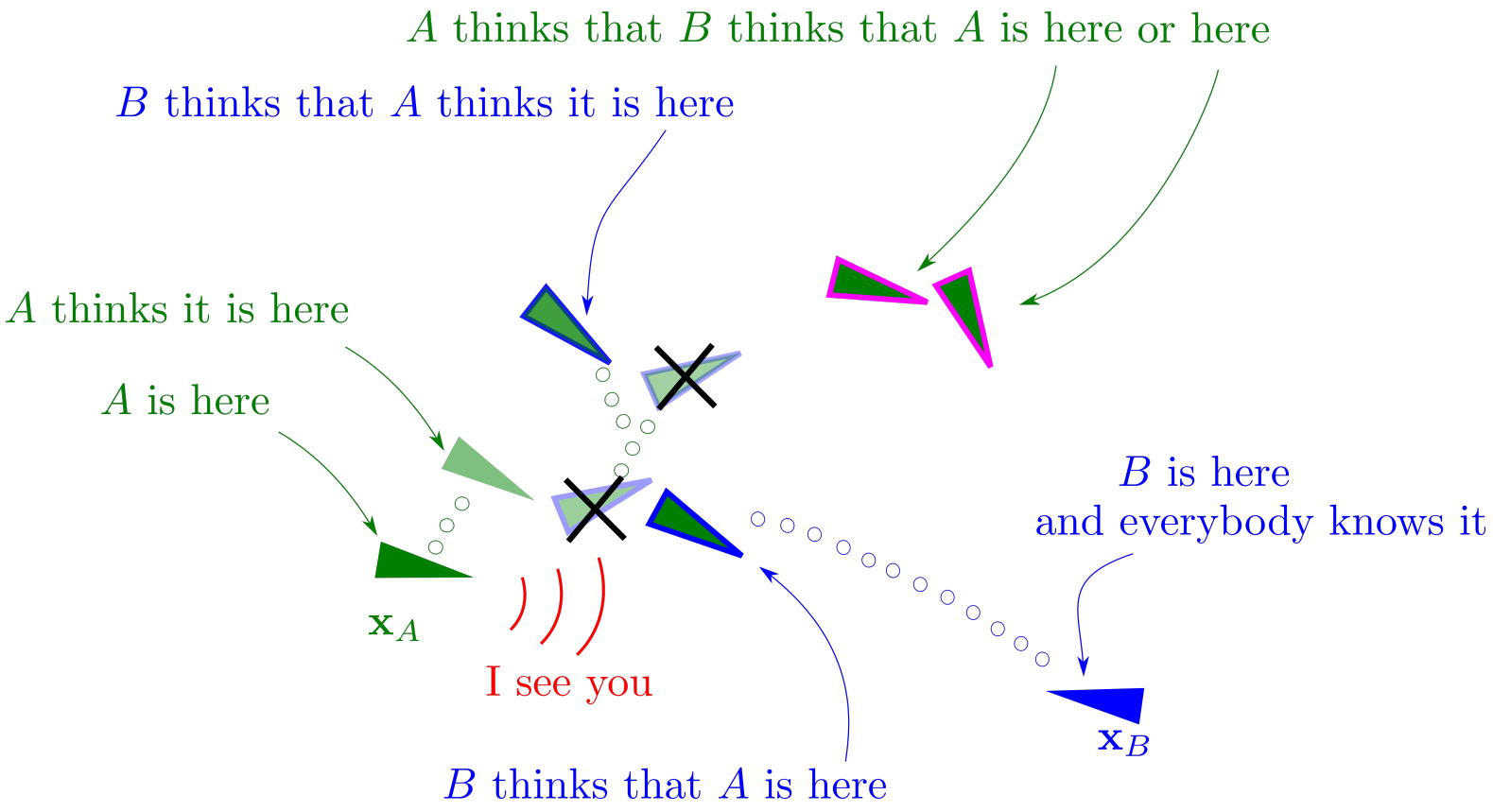

A thinks that B thinks that A is here
B thinks that A thinks it is here
A thinks it is here
A is here
B thinks it is here
B is here
$\mathbf{x}_A$
$\mathbf{x}_B$
B thinks that A is here

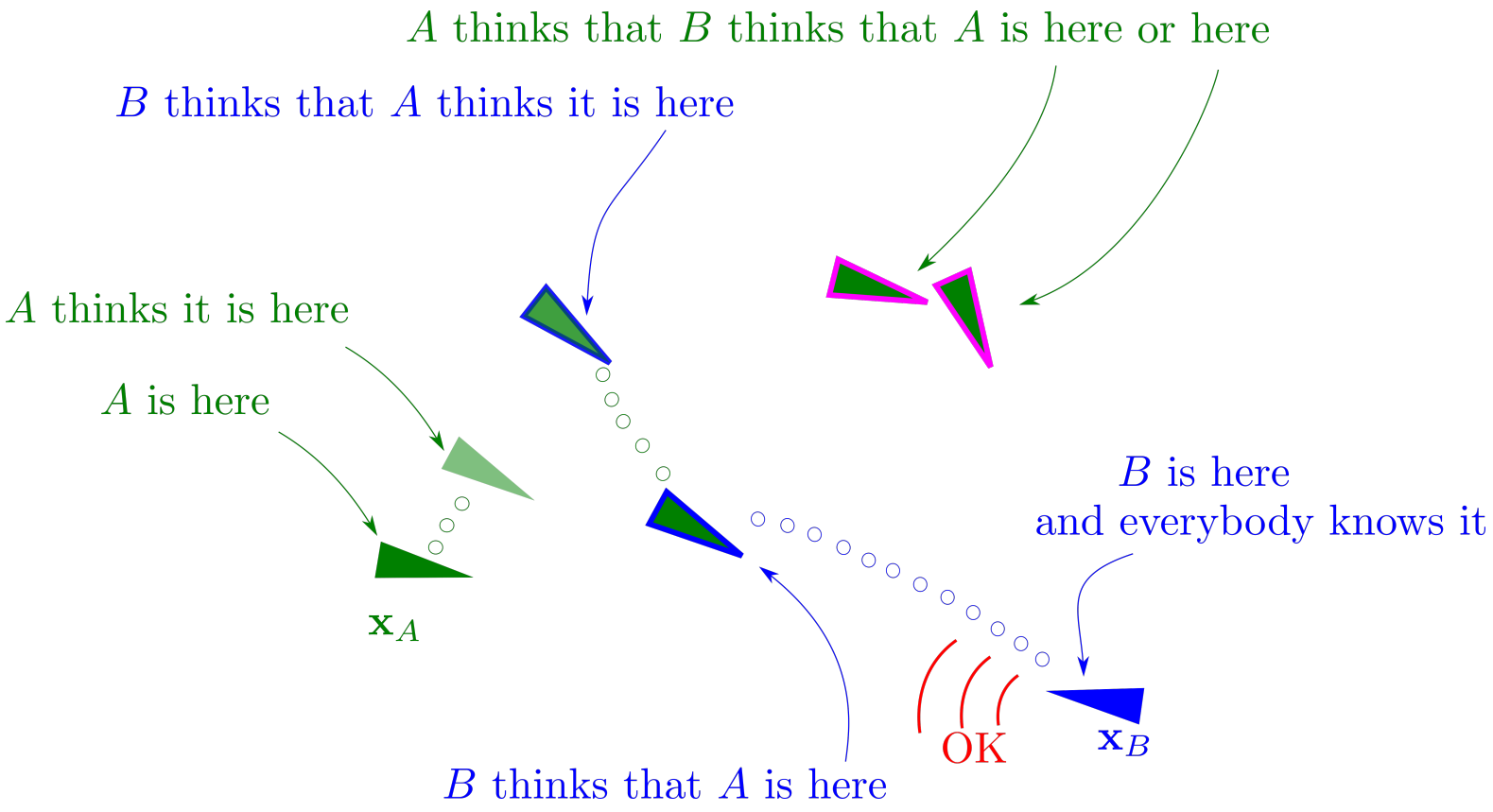
Communication

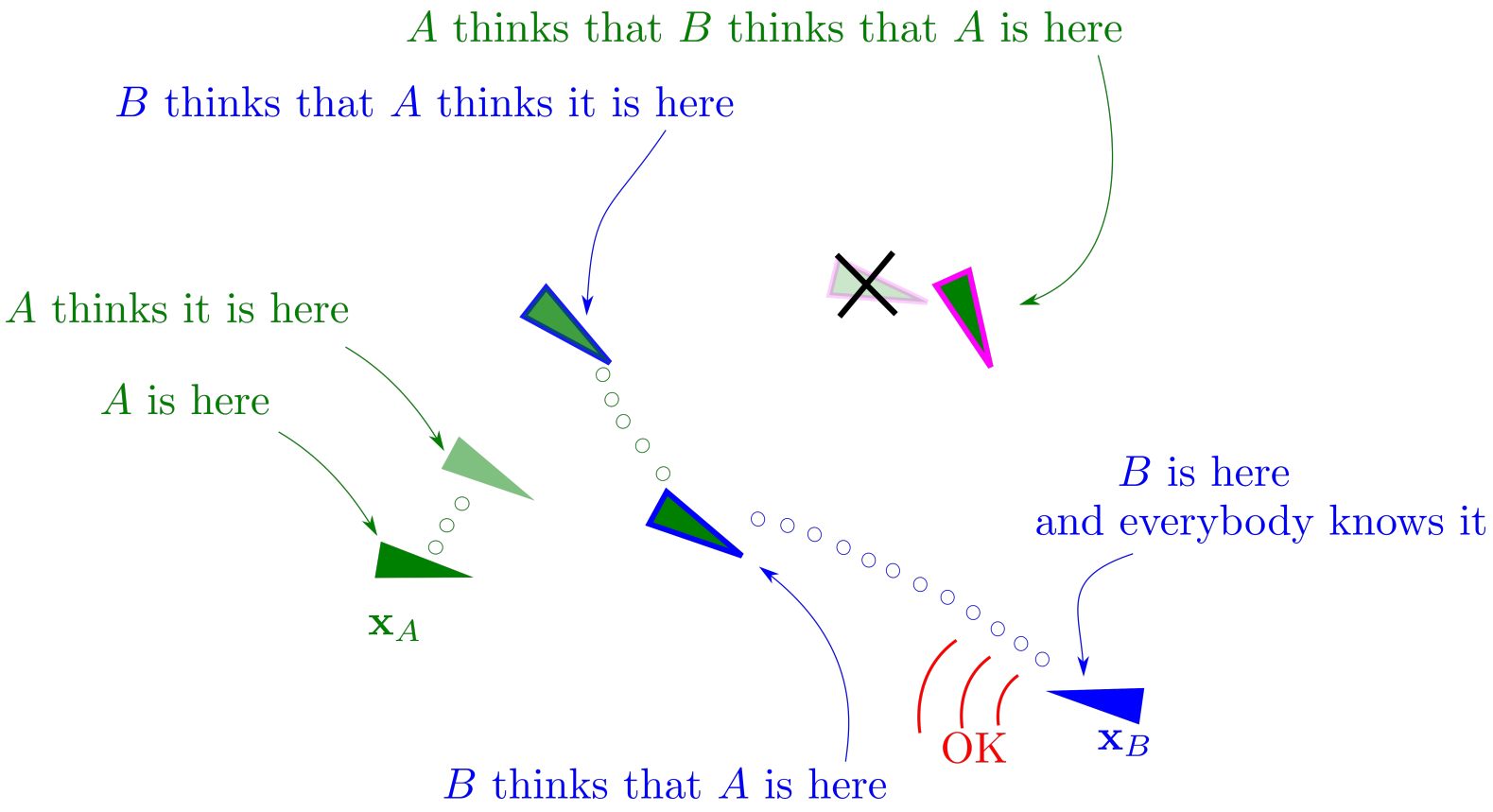
A communication is a measure of the knowledge.


A thinks that B thinks that A is here or here
B thinks that A thinks it is here or here
A thinks it is here
A is here
B is here and everybody knows it
$\mathbf{x}_A$
$\mathbf{x}_B$
B thinks that A is here or here


A thinks that B thinks that A is here or here
B thinks that A thinks it is here or here
A thinks it is here
A is here
B is here and everybody knows it
$\mathbf{x}_A$
I see you
$\mathbf{x}_B$
B thinks that A is here or here


A thinks that B thinks that A is here or here
B thinks that A thinks it is here
A thinks it is here
A is here
B is here
and everybody knows it
$\mathbf{x}_A$
I see you
$\mathbf{x}_B$
B thinks that A is here


A thinks that B thinks that A is here or here
B thinks that A thinks it is here
A thinks it is here
A is here
B is here
and everybody knows it
$\mathbf{x}_A$
OK
$\mathbf{x}_B$
B thinks that A is here


A thinks that B thinks that A is here
B thinks that A thinks it is here
A thinks it is here
A is here
$\mathbf{x}_A$
B is here
and everybody knows it
OK
$\mathbf{x}_B$
B thinks that A is here

References

1. Dynamic Epistemic Logic : [2]
2. Swarm localization [1]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