SLAM with magnetism only

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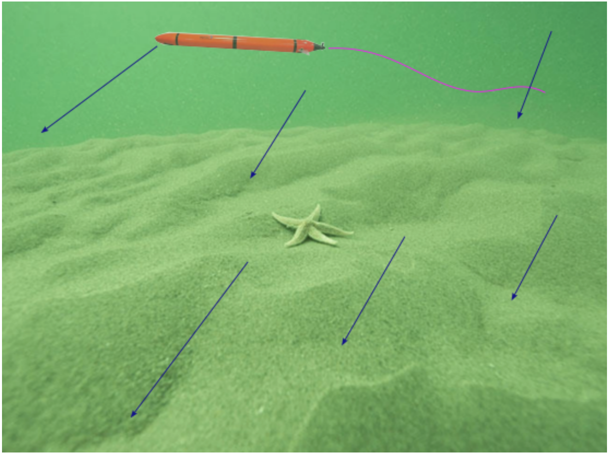
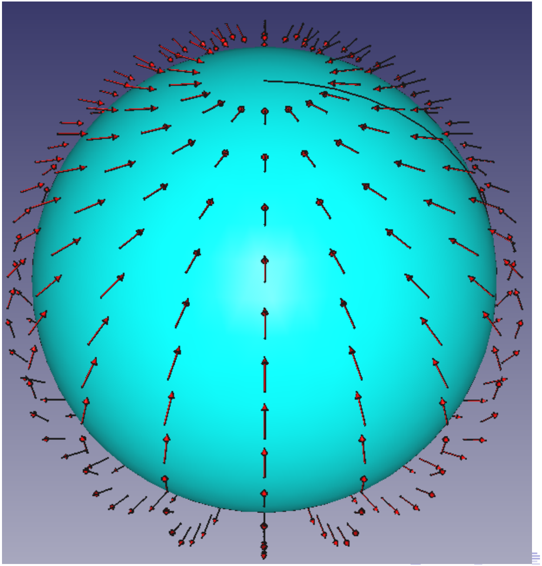
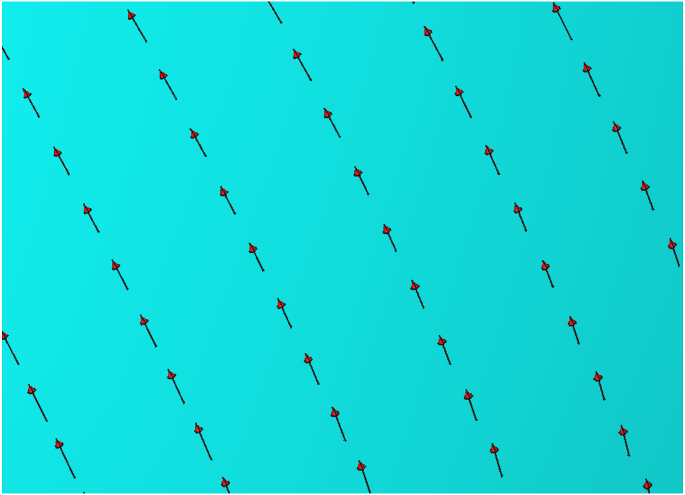
February 5, 2026, CHRoMM meeting, ENSTA Palaiseau

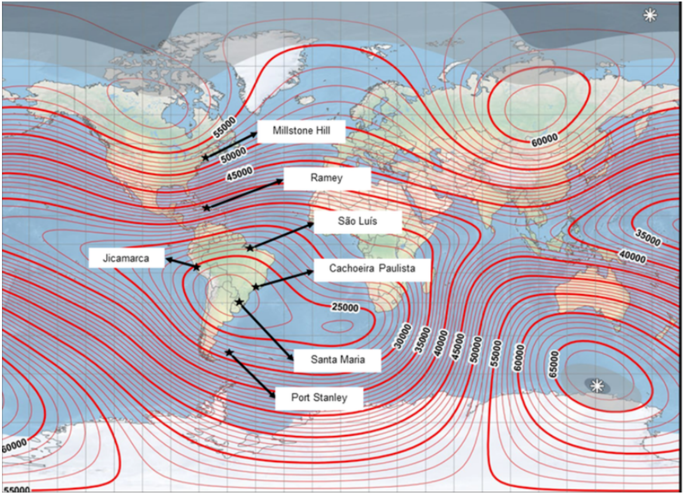


Explore and build a map in a magnetic environment

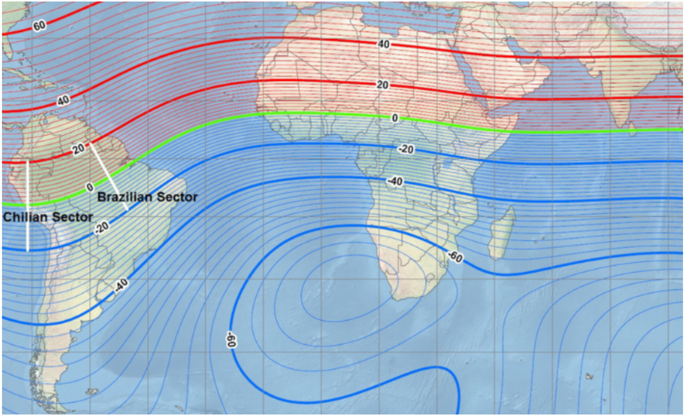
1. Magnetic world





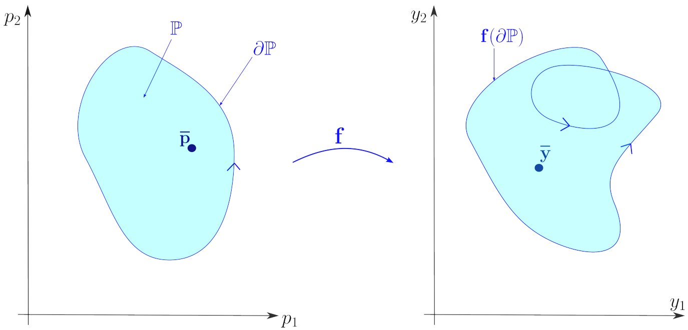


Intensity [nT]

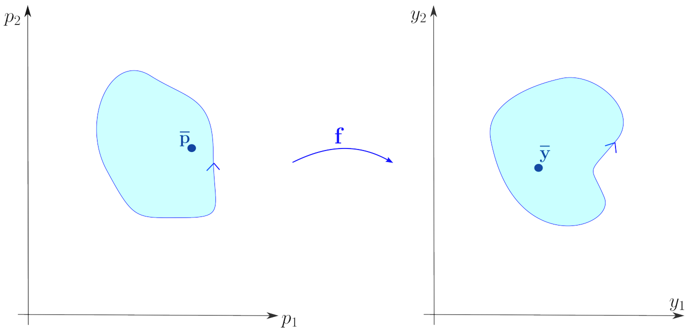


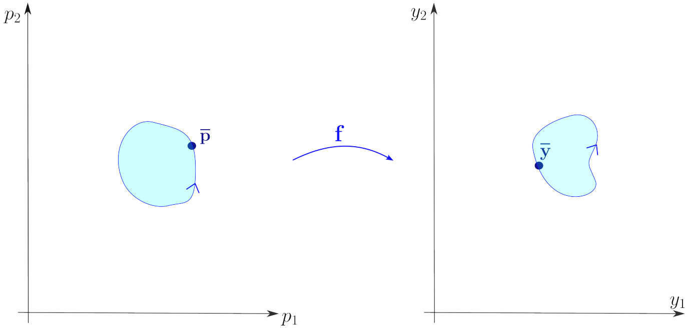
Inclinaison [deg]

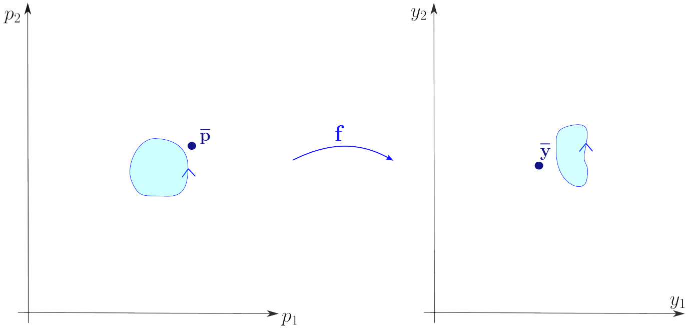
2. Topological degree



$$\deg(\mathbf{f}, \mathbb{P}, \bar{\mathbf{y}}) \neq 0 \Rightarrow \exists \bar{\mathbf{p}} \in \mathbb{P} \mid \mathbf{f}(\bar{\mathbf{p}}) = \bar{\mathbf{y}}$$


p_2
$\bar{\mathbf{p}}$
p_1
$\mathbf{f}$
y_2
$\bar{\mathbf{y}}$
y_1


p_2
$\bar{\mathbf{p}}$
p_1
$\mathbf{f}$
y_2
$\bar{\mathbf{y}}$
y_1


p_2
$\bar{\mathbf{p}}$
p_1
$\mathbf{f}$
y_2
$\bar{\mathbf{y}}$
y_1

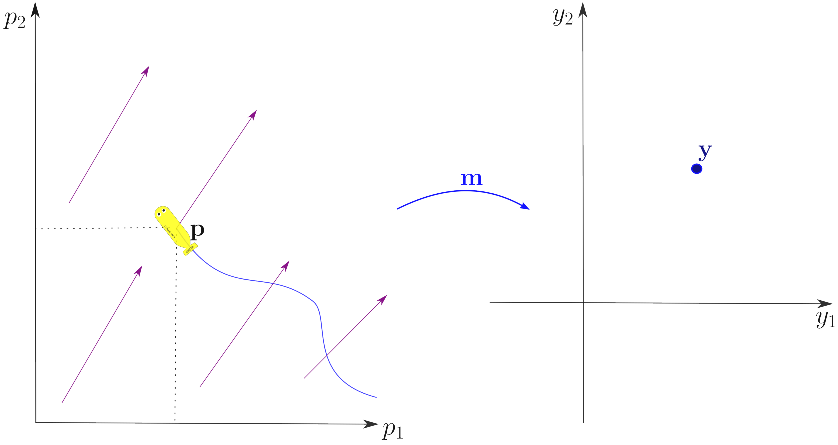
3. Mag-based navigation

$$\begin{cases} \dot{p}_1 &= q_2 \cdot \cos q_1 \\ \dot{p}_2 &= q_2 \cdot \sin q_1 \\ \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \end{cases} \quad \text{and} \quad \begin{cases} y_1 &= m_1(\mathbf{p}) & \text{(inclinaison)} \\ y_2 &= m_2(\mathbf{p}) & \text{(intensity)} \end{cases}$$

Or more generally

$$\begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \end{pmatrix} = \mathbf{f}(\mathbf{p},\mathbf{q},\mathbf{u}) \text{ and } \begin{cases} y_1 = m_1(\mathbf{p}) & \text{(inclinaison)} \\ y_2 = m_2(\mathbf{p}) & \text{(intensity)} \end{cases}$$

where $\mathbf{p} \in \mathbb{R}^2$ is the position.

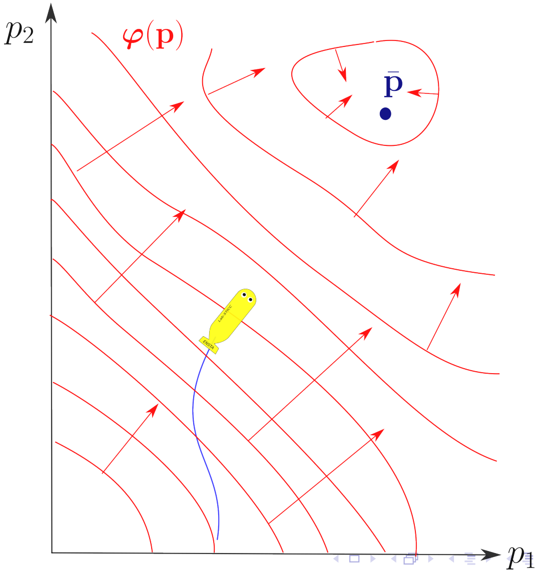

p_2
p_1
$\mathbf{p}$
$\mathbf{m}$
y_2
y_1
$\mathbf{y}$

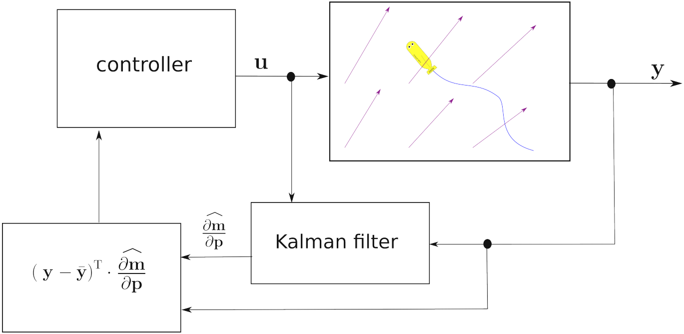
Go home

The robot wants to go home, *i.e.*, to a position $\bar{\mathbf{p}}$ where $\bar{\mathbf{y}} = \mathbf{m}(\bar{\mathbf{p}})$.
Think of the pigeon who returns to its loft using mag only.
It can move in order to minimize $\|\mathbf{m}(\mathbf{p}) - \bar{\mathbf{y}}\|^2$

Strategies: We build a clever field $\varphi(\mathbf{p})$ to follow.
For instance, for a *reach home*, we take

$$\begin{aligned}\varphi(\mathbf{p}) &= \frac{\partial}{\partial \mathbf{p}}\|\mathbf{m}(\mathbf{p})-\bar{\mathbf{y}}\|^2 \\ &= 2\left(\mathbf{m}(\mathbf{p})-\bar{\mathbf{y}}\right)^{\mathsf{T}}\cdot\frac{\partial \mathbf{m}}{\partial \mathbf{p}}(\mathbf{p})\end{aligned}$$


p_2
$\varphi(\mathbf{p})$
$\bar{\mathbf{p}}$
p_1



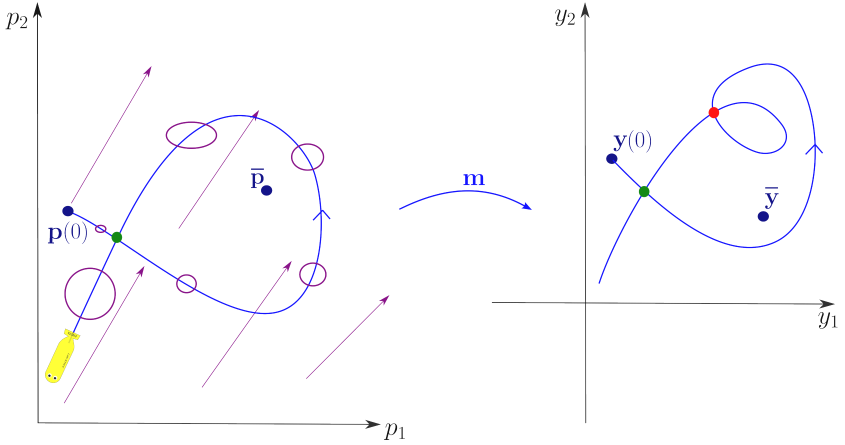
We need to estimate the variation $\frac{\partial \mathbf{m}}{\partial \mathbf{p}}$ using a Kalman filter

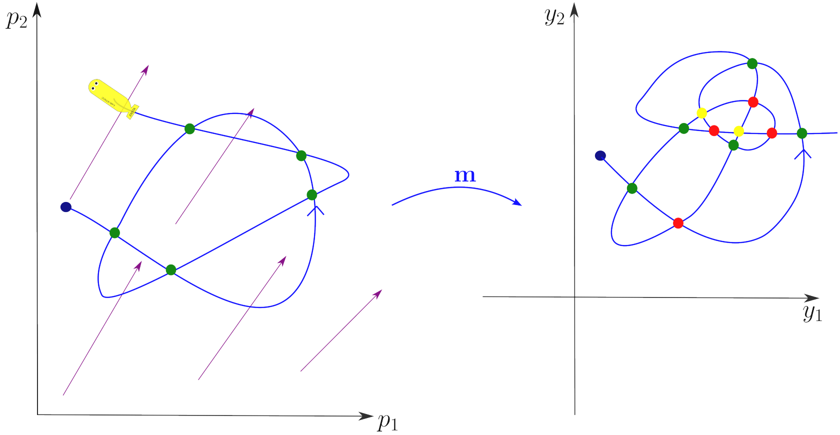
Unfortunately , we may converge to a local minimizer.

For SLAM, we need loops

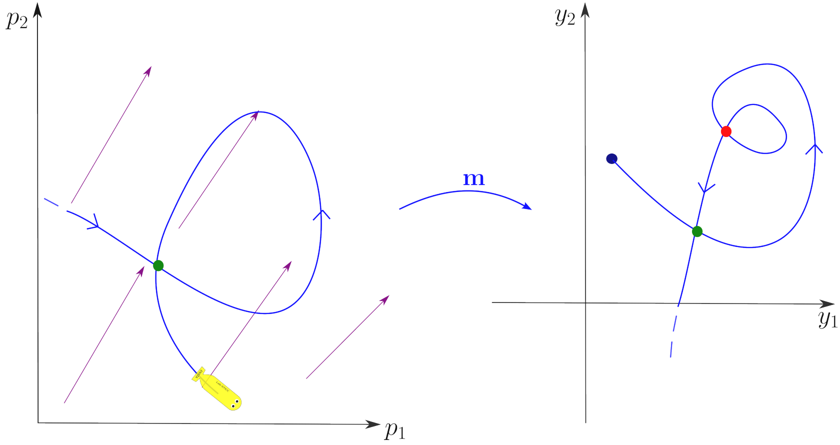
We are able

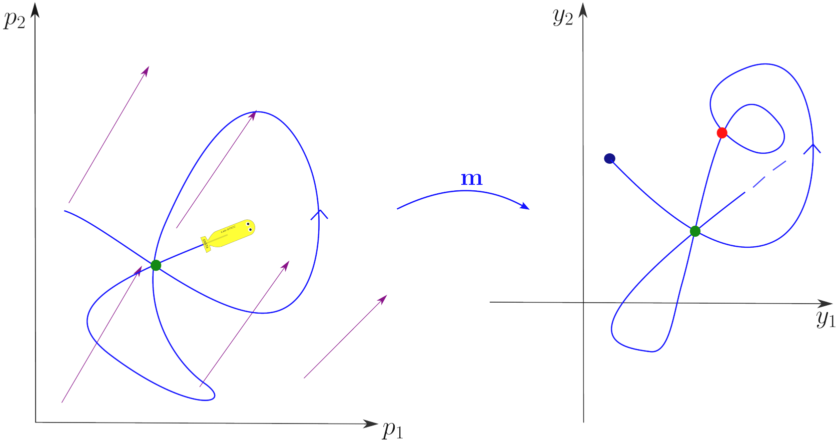
- to detect and prove the existence of a loop
- to prove the existence of $\mathbf{p}$ such that $\bar{\mathbf{y}} = \mathbf{m}(\mathbf{p})$ (topological degree)


p_2
p_1
$\bar{\mathbf{p}}$
$\mathbf{p}(0)$
$\mathbf{m}$
y_2
y_1
$\mathbf{y}(0)$
$\bar{\mathbf{y}}$

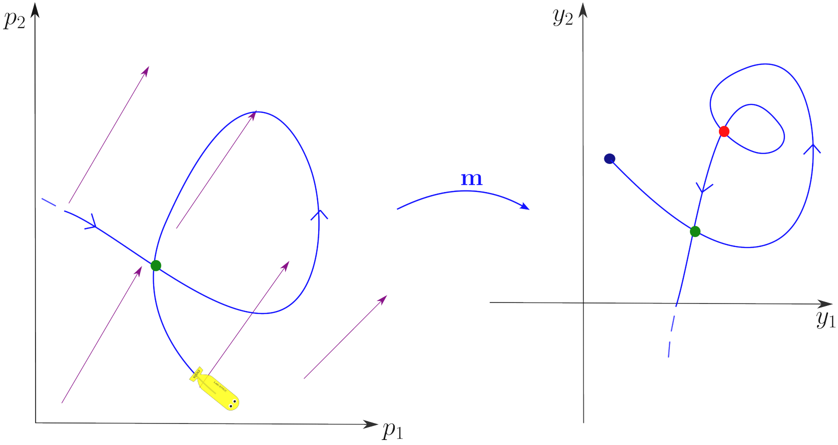

p_2
p_1
m
y_2
y_1

Back to a past loop

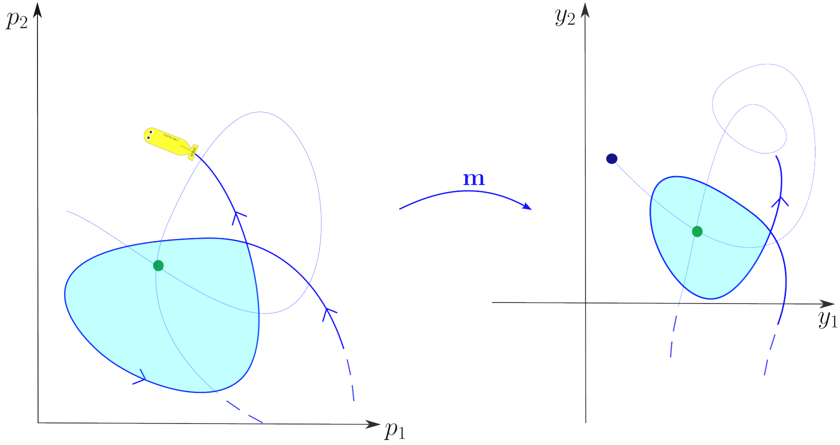

p_2
p_1
m
y_2
y_1

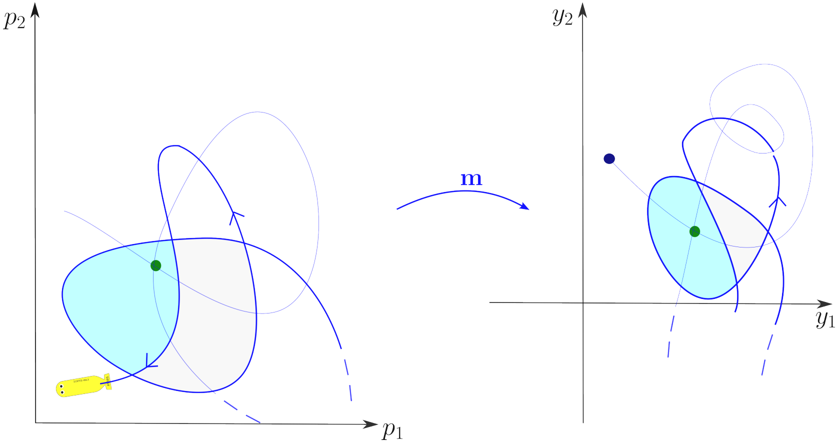


A three loop is impossible

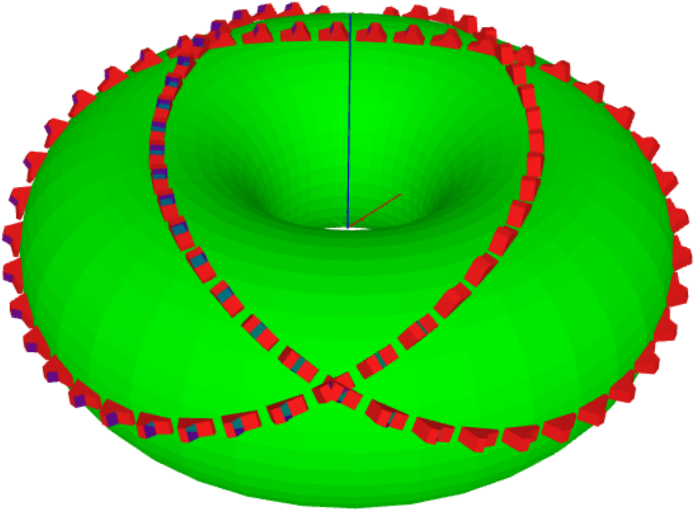


We need the topological degree.

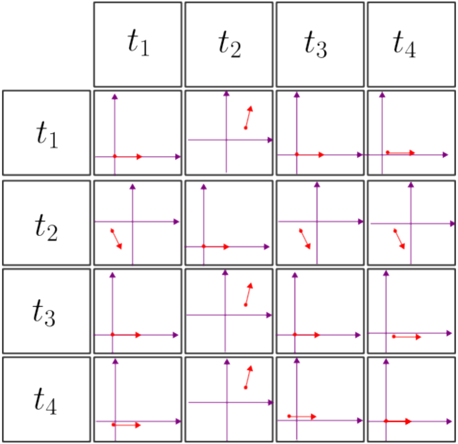

p_2
p_1
$\mathbf{m}$
y_2
y_1


p_2
p_1
$\mathbf{m}$
y_2
y_1

Toward a metric map: the pose matrix



We copy the representation by distance matrix.
The landmarks (nodes) are loop points.
They are associated to the north unit vector.
Nodes are linked by a pose (translation + rotation).

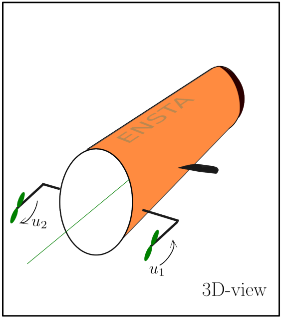


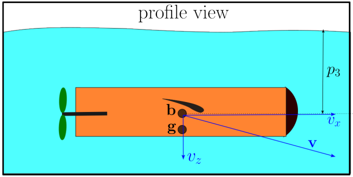
One true loop (t_1, t_3), two almost loops $(t_1, t_4), (t_3, t_4)$,

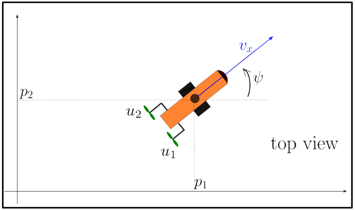
No loop with t_2

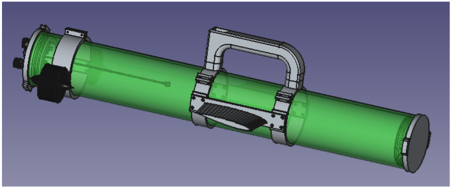
4. Challenge (March 4, 2026)

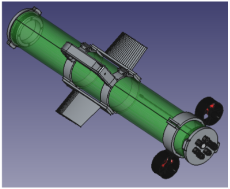
Subsurface


ENSTA
u_2
u_1
3D-view


profile view
p_3
b
g
v_x
v
v_z


v_x
ψ
p_2
u_2
u_1
top view
p_1





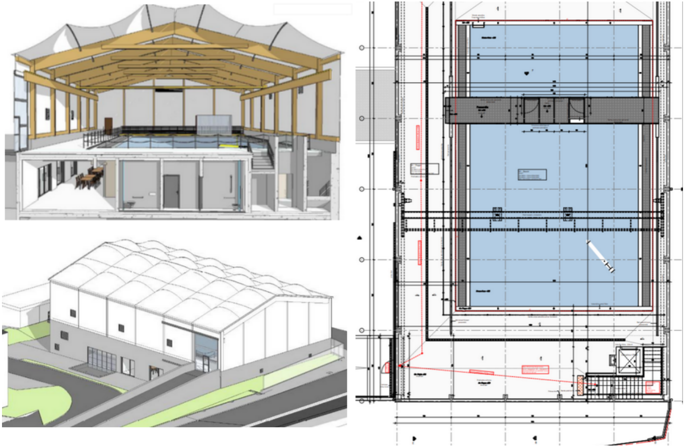
Mechanics

- The robot has two actuators, here propellers to move forward or turn horizontally
- The robot has a positive buoyancy, meaning that the resultant of the Archimedes force and the weight is a constant vertical force f_a oriented upward
- Two wings located laterally exert a force downward
- The center of gravity $\mathbf{g}$ is just below the center of volume $\mathbf{b}$.

Sensors

The robot has

- a GPS available in the pool
- A pressure sensor
- An IMU (magnetometer, gyro, accelerometer)



Bassin robotique

All missions have be done autonomously.

- **Challenge 1**. Immersion and surface. The robot has to start at the surface point $\mathbf{a}$, dive 1 meter deep and then has to come back to the surface at $\mathbf{b}$
- **Challenge 2**. Go from $\mathbf{a}$ to $\mathbf{b}$, then from $\mathbf{b}$ to $\mathbf{a}$, and repeat as many times as possible. Between $\mathbf{a}$ and $\mathbf{b}$, the robot should be underwater.
- **Challenge 3**. Perform a stable cycle, underwater. No GPS can be used. Magnetism only

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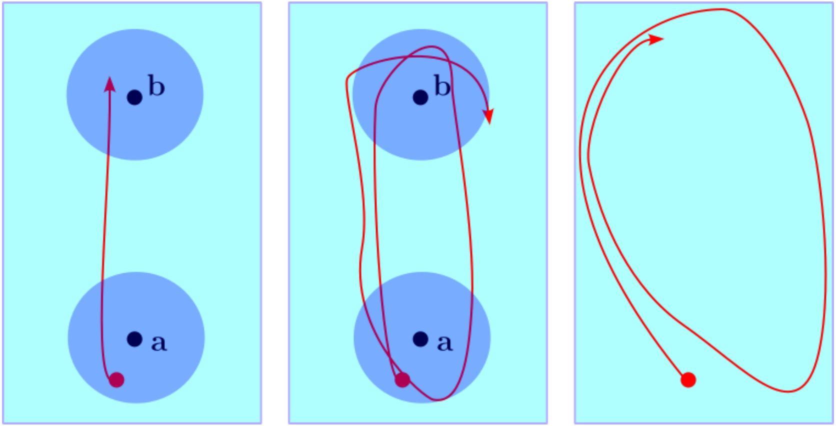

Challenges 1,2 and 3

Stable cycles add determinism and may allow a new way to solve the Rendez-vous problem.

References

1. Navigation with stable cycles [2]
2. Loops [4][1]
3. Topological degree and exploration [6][5]
4. Brouwer for loop closure [3]

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