

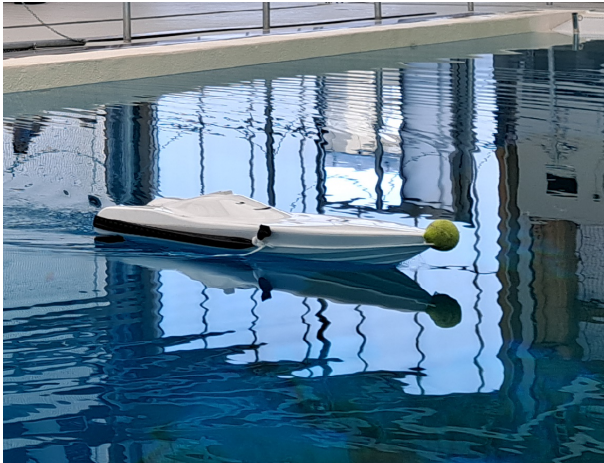
A $(max, +)$ based interval contractor for swarm localization

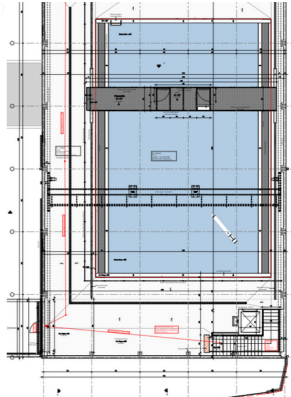
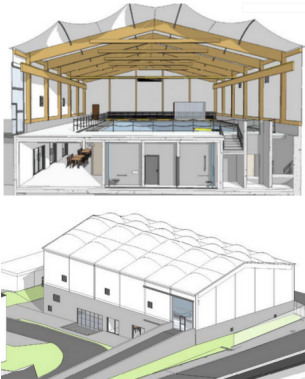
Luc Jaulin and Laurent Hardouin

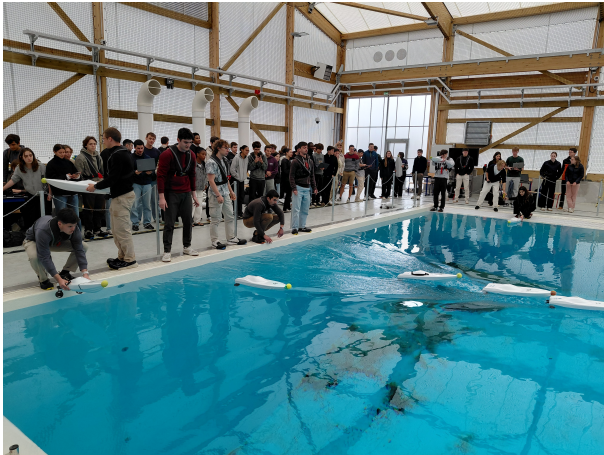


Kraków, Poland June, 2026, SWIM'2026

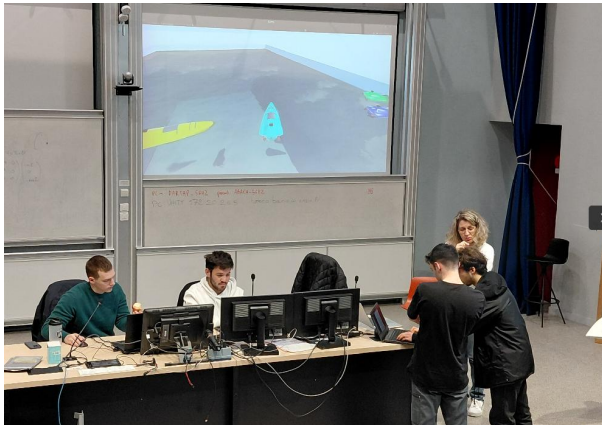
1. Differential GPS

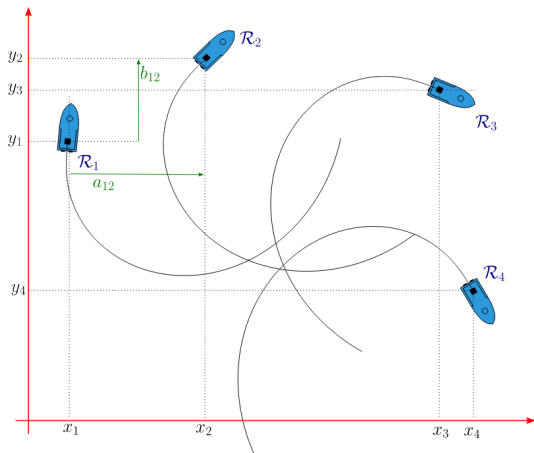












(x_i, y_i) is known with a poor accuracy compared to the a_{ij}, b_{ij}

Consider m robots $\mathcal{R}_i$, $i \in \{1, 2, \dots, m\}$:

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{v}_i \end{pmatrix} = \begin{pmatrix} v_i \cos \theta_i \\ v_i \sin \theta_i \\ \omega_i \\ \alpha_i \end{pmatrix}$$

where θ_i is its heading and v_i is the speed.

The inputs are the yaw ω_i and the acceleration α_i .

For each $t_k = k \cdot dt_k$, $k \geq 0$,

- each $\mathcal{R}_i$ measures $(x_i(k), y_i(k))$ with a precision $\pm e_{GPS}$. For each t_k , we have $\sim m$ intervals $[x_i]$ and $[y_i]$.
- for any pair of robots (i, j) , the vector

$$\begin{pmatrix} a_{ij}(k) \\ b_{ij}(k) \end{pmatrix} = \begin{pmatrix} x_j(k) - x_i(k) \\ y_j(k) - y_i(k) \end{pmatrix}$$

is measured with an error of $\pm e_{DGPS}$. For every t_k , we have $\sim m^2$ intervals $[a_{ij}]$ and $[b_{ij}]$.

For each (i,j,ℓ) , we have $\sim O(m^3)$ equations

$$a_{ij} = a_{i\ell} + a_{\ell j} \quad (\text{Chasles})$$

$$b_{ij} = b_{i\ell} + b_{\ell j}$$

$$a_{ij} + a_{ji} = 0 \quad (\text{symmetry})$$

$$b_{ij} + b_{ji} = 0$$

These relations can be used to contract the intervals for a_{ij}, b_{ij} .

The propagation could be long.

We would like to do better considering all these constraints as a global constraint.

We will build a **fixed-point contractor** using **$(\max, +)$ algebra**.

2. $(\max, +)$ algebra

$$\begin{cases} a \oplus b = \max(a, b) \\ a \otimes b = a + b \end{cases}$$

Example:

$$2 \oplus 3 = ?$$

$$2 \otimes 3 = ?$$

$$\begin{cases} a \oplus b = \max(a, b) \\ a \otimes b = a + b \end{cases}$$

Example:

$$2 \oplus 3 = 3$$

$$2 \otimes 3 = 5$$

- $(\mathbb{R}, \oplus)$ is not a group, only a monoid.
- $(\mathbb{R}, \oplus, \otimes)$ is not a ring only a dioid (tropical semiring).
- We can build a matrix algebra

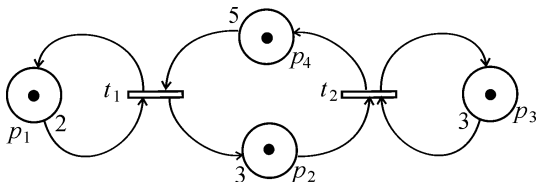
Why $(\max, +)$ algebra?

Bus 1 left Rennes at time x_1 . It takes 2 hours to arrive to Brest.
Bus 2 left Paris at time x_2 . It takes 5 hours to arrive to Brest.
When both buses are in Brest, Bus 1 will leave Brest to go to Quimper.

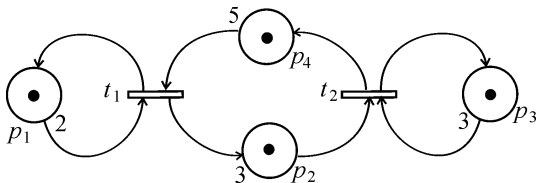
At what time Bus 1 will leave ?

Bus 1 can leave at times:

$$\begin{aligned}x_3 &= \max(x_1 + 2, x_2 + 5) \\&= 2 \otimes x_1 \oplus 5 \otimes x_2 \\&= \begin{pmatrix} 2 & 5 \end{pmatrix} \otimes \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\end{aligned}$$



$$\begin{cases} x_1(k+1) &= \max(x_1(k) + 2, x_2(k) + 5) \\ x_2(k+1) &= \max(x_1(k) + 3, x_2(k) + 3) \end{cases}$$



$$\mathbf{x}(k+1) = \begin{pmatrix} 2 & 5 \\ 3 & 3 \end{pmatrix} \otimes \mathbf{x}(k)$$

A matrix $\mathbf{A}$ is said to be $\otimes$ *idempotent* if

$$\mathbf{A}^{\otimes 2} = \mathbf{A}.$$

Example. The matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$$

is idempotent:

$$\begin{aligned} & \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \\ = & \begin{pmatrix} \max(0+0, 1-2) & \max(0+1, 1+0) \\ \max(-2+0, 0-2) & \max(-2+1, 0+0) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \end{aligned}$$

In the tropical algebra, the identity matrix is

$$\mathbf{I} = \begin{pmatrix} 0 & -\infty & \dots & -\infty \\ -\infty & 0 & \vdots & \vdots \\ \vdots & \vdots & \ddots & -\infty \\ -\infty & \dots & -\infty & 0 \end{pmatrix}$$

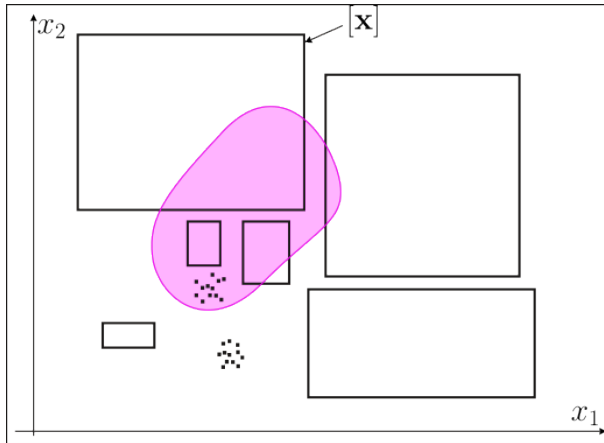
3. Fixed-point contractors

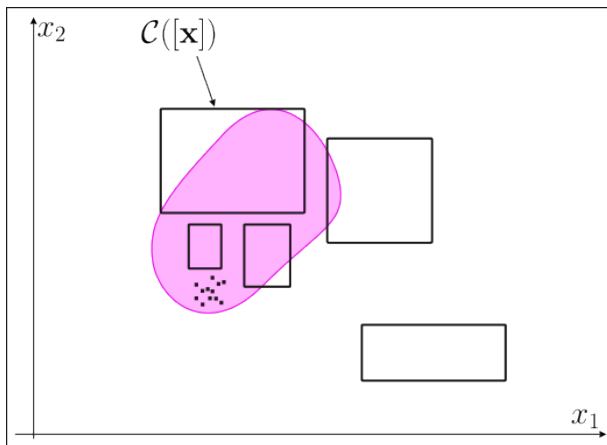
A mapping $\mathcal{C}:\mathbb{R}^n \mapsto \mathbb{R}^n$ is a *contractor* for a set $\mathbb{X} \subset \mathbb{R}^n$ if:

$$\mathcal{C}([\mathbf{x}]) \subset [\mathbf{x}] \quad (\text{consistency})$$

$$[\mathbf{x}] \cap \mathbb{X} \subset \mathcal{C}([\mathbf{x}]) \quad (\text{correctness})$$

$$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}([\mathbf{x}]) \subset \mathcal{C}([\mathbf{y}]) \quad (\text{monotonicity})$$





To build a fixed-point contractor for

$$\mathbb{X} = \{\mathbf{x} \mid \mathbf{f}(\mathbf{x}) = \mathbf{0}\}$$

we rewrite the equation into a *fixed point constraint*;

$$\mathbf{g}(\mathbf{x}) = \mathbf{x}.$$

A fixed-point contractor is given by

$$\mathcal{C}([\mathbf{x}]) = [\mathbf{x}] \cap [\mathbf{g}]([\mathbf{x}])$$

where $[\mathbf{g}]$ is an interval extension of $\mathbf{g}$.

Example. The equation

$$x_1 + x_2 = x_3$$

is equivalent to:

$$\underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{\mathbf{g}(\mathbf{x})} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

The corresponding fixed-point contractor is

$$\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} \mapsto \begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} \cap \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix}$$

Example. The equation

$$x_1 \oplus x_2 = x_3$$

is equivalent to:

$$\underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} x_1 \ominus x_3 \\ x_2 \ominus x_3 \\ x_1 \oplus x_2 \end{pmatrix}}_{\mathbf{g}(\mathbf{x})}$$

where $\oplus$ and $\ominus$ denote the max and min operators.

A fixed-point contractor is given by

$$\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} \mapsto \begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} \cap \begin{pmatrix} [x_1] \ominus [x_3] \\ [x_2] \ominus [x_3] \\ [x_1] \oplus [x_2] \end{pmatrix}$$

For matrices. A contractor “the matrix $\mathbf{A}$ is idempotent” , *i.e.*, $\mathbf{A}^2 = \mathbf{A}$ is:

$$\mathcal{C}([\mathbf{A}]) = [\mathbf{A}] \cap [\mathbf{A}]^2.$$

Differential constraints: The state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, rewrites into a fixed point form:

$$\mathbf{x}(t) = \mathbf{x}(0) + \int_0^t \mathbf{f}(\mathbf{x}(\tau)) \cdot d\tau$$

known as the Picard equation. The associated fixed-point contractor is

$$[\mathbf{x}](t) = [\mathbf{x}](t) \cap \left([\mathbf{x}](0) + \int_0^t [\mathbf{f}]([\mathbf{x}](\tau)) \cdot d\tau \right).$$

4. Contractor for potential matrices

A *potential matrix* is a matrix $\mathbf{A}$ with entries a_{ij} which satisfies

$$a_{ij} = a_{ik} + a_{kj} \quad (\text{Chasles})$$

$$a_{ij} + a_{ji} = 0 \quad (\text{skew symmetry})$$

A potential matrix encompasses the DGPS constraints.

We have

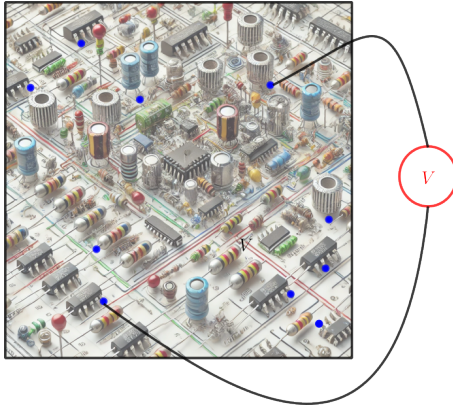
$$\mathbf{A} \text{ is potential} \Leftrightarrow \begin{cases} \mathbf{A} &= \mathbf{A} \oplus \mathbf{I} \\ \mathbf{A} &= \mathbf{A}^{\otimes 2} \\ \mathbf{A} &= -\mathbf{A}^T \end{cases}$$

A contractor for the constraint “ $\mathbf{A}$ is a potential matrix”, is given by

$$\mathcal{C}_{\text{pot}}([\mathbf{A}]) = [\mathbf{A}] \cap ([\mathbf{A}] \oplus \mathbf{I}) \cap [\mathbf{A}]^{\otimes 2} \cap -[\mathbf{A}]^{\text{T}}.$$

Moreover, a $\mathcal{C}_{\text{pot}}^{\infty}$ is minimal.

Example: electric circuit



We measure n^2 voltage a_{ij} between each pair of n nodes

We collected the following interval matrix

$$[\mathbf{A}] = \begin{pmatrix} [0, 4] & [12, 16] & [6, 10] & [4, 8] & [-2, 2] & [13, 17] & [-3, 1] & [11, 15] & [4, 8] & [0, 4] \\ [-14, -10] & [-3, 1] & [-5, -1] & [-10, -6] & [-16, -12] & [-3, 1] & [-15, -11] & [-3, 1] & [-12, -8] & [-10, -6] \\ [-9, -5] & [3, 7] & [-1, 3] & [-5, -1] & [-11, -7] & [4, 8] & [-8, -4] & [2, 6] & [-4, 0] & [-8, -4] \\ [-7, -3] & [8, 12] & [0, 4] & [-1, 3] & [-6, -2] & [7, 11] & [-8, -4] & [5, 9] & [0, 4] & [-3, 1] \\ [-2, 2] & [11, 15] & [6, 10] & [2, 6] & [0, 4] & [11, 15] & [-5, -1] & [10, 14] & [0, 4] & [2, 6] \\ [-17, -13] & [-4, 0] & [-7, -3] & [-10, -6] & [-16, -12] & [-3, 1] & [-15, -11] & [-5, -1] & [-13, -9] & [-11, -7] \\ [-1, 3] & [9, 13] & [8, 12] & [4, 8] & [-2, 2] & [14, 18] & [-3, 1] & [8, 12] & [3, 7] & [3, 7] \\ [-13, -9] & [1, 5] & [-8, -4] & [-8, -4] & [-11, -7] & [-2, 2] & [-16, -12] & [-4, 0] & [-12, -8] & [-9, -5] \\ [-6, -2] & [7, 11] & [0, 4] & [-2, 2] & [-5, -1] & [6, 10] & [-6, -2] & [8, 12] & [-3, 1] & [-3, 1] \\ [-8, -4] & [8, 12] & [0, 4] & [-1, 3] & [-6, -2] & [6, 10] & [-4, 0] & [6, 10] & [-1, 3] & [-2, 2] \end{pmatrix}$$

$$C_{\text{pot}}(\mathbf{A}) = \begin{pmatrix} [0, 0] & [12, 14] & [7, 9] & [4, 6] & [0, 2] & [13, 16] & [-2, 1] & [12, 13] & [4, 5] & [4, 4] \\ [-14, -12] & [0, 0] & [-5, -3] & [-10, -8] & [-14, -12] & [0, 1] & [-13, -12] & [-3, -1] & [-9, -9] & [-10, -8] \\ [-9, -7] & [3, 5] & [0, 0] & [-4, -3] & [-9, -7] & [6, 7] & [-8, -8] & [4, 4] & [-4, -2] & [-4, -4] \\ [-6, -4] & [8, 10] & [3, 4] & [0, 0] & [-5, -2] & [7, 10] & [-6, -4] & [8, 8] & [0, 0] & [-2, 0] \\ [-2, 0] & [12, 14] & [7, 9] & [2, 5] & [0, 0] & [12, 15] & [-2, -1] & [10, 11] & [2, 3] & [2, 5] \\ [-16, -13] & [-1, 0] & [-7, -6] & [-10, -7] & [-15, -12] & [0, 0] & [-15, -14] & [-2, -1] & [-10, -9] & [-10, -10] \\ [-1, 2] & [12, 13] & [8, 8] & [4, 6] & [1, 2] & [14, 15] & [0, 0] & [12, 12] & [4, 6] & [3, 4] \\ [-13, -12] & [1, 3] & [-4, -4] & [-8, -8] & [-11, -10] & [1, 2] & [-12, -12] & [0, 0] & [-8, -8] & [-9, -7] \\ [-5, -4] & [9, 9] & [2, 4] & [0, 0] & [-3, -2] & [9, 10] & [-6, -4] & [8, 8] & [0, 0] & [-1, 1] \\ [-4, -4] & [8, 10] & [4, 4] & [0, 2] & [-5, -2] & [10, 10] & [-4, -3] & [7, 9] & [-1, 1] & [0, 0] \end{pmatrix}$$

$$C_{\text{pot}}^2([A]) = \begin{pmatrix} [0,0] & [13,14] & [8,8] & [4,5] & [1,2] & [14,14] & [0,1] & [12,13] & [4,5] & [4,4] \\ [-14,-13] & [0,0] & [-5,-5] & [-9,-9] & [-12,-12] & [1,1] & [-13,-13] & [-1,-1] & [-9,-9] & [-9,-9] \\ [-8,-8] & [5,5] & [0,0] & [-4,-4] & [-7,-7] & [6,6] & [-8,-8] & [4,4] & [-4,-4] & [-4,-4] \\ [-5,-4] & [9,9] & [4,4] & [0,0] & [-3,-3] & [9,10] & [-4,-4] & [8,8] & [0,0] & [-1,0] \\ [-2,-1] & [12,12] & [7,7] & [3,3] & [0,0] & [13,13] & [-1,-1] & [11,11] & [3,3] & [3,3] \\ [-14,-14] & [-1,-1] & [-6,-6] & [-10,-9] & [-13,-13] & [0,0] & [-14,-14] & [-2,-2] & [-10,-9] & [-10,-10] \\ [-1,0] & [13,13] & [8,8] & [4,4] & [1,1] & [14,14] & [0,0] & [12,12] & [4,4] & [4,4] \\ [-13,-12] & [1,1] & [-4,-4] & [-8,-8] & [-11,-11] & [2,2] & [-12,-12] & [0,0] & [-8,-8] & [-8,-8] \\ [-5,-4] & [9,9] & [4,4] & [0,0] & [-3,-3] & [9,10] & [-4,-4] & [8,8] & [0,0] & [-1,0] \\ [-4,-4] & [9,9] & [4,4] & [0,1] & [-3,-3] & [10,10] & [-4,-4] & [8,8] & [0,1] & [0,0] \end{pmatrix}$$

$$C_{\text{pot}}^3([A]) = \begin{pmatrix} [0, 0] & [13, 13] & [8, 8] & [4, 4] & [1, 1] & [14, 14] & [0, 0] & [12, 12] & [4, 4] & [4, 4] \\ [-13, -13] & [0, 0] & [-5, -5] & [-9, -9] & [-12, -12] & [1, 1] & [-13, -13] & [-1, -1] & [-9, -9] & [-9, -9] \\ [-8, -8] & [5, 5] & [0, 0] & [-4, -4] & [-7, -7] & [6, 6] & [-8, -8] & [4, 4] & [-4, -4] & [-4, -4] \\ [-4, -4] & [9, 9] & [4, 4] & [0, 0] & [-3, -3] & [10, 10] & [-4, -4] & [8, 8] & [0, 0] & [0, 0] \\ [-1, -1] & [12, 12] & [7, 7] & [3, 3] & [0, 0] & [13, 13] & [-1, -1] & [11, 11] & [3, 3] & [3, 3] \\ [-14, -14] & [-1, -1] & [-6, -6] & [-10, -10] & [-13, -13] & [0, 0] & [-14, -14] & [-2, -2] & [-10, -10] & [-10, -10] \\ [0, 0] & [13, 13] & [8, 8] & [4, 4] & [1, 1] & [14, 14] & [0, 0] & [12, 12] & [4, 4] & [4, 4] \\ [-12, -12] & [1, 1] & [-4, -4] & [-8, -8] & [-11, -11] & [2, 2] & [-12, -12] & [0, 0] & [-8, -8] & [-8, -8] \\ [-4, -4] & [9, 9] & [4, 4] & [0, 0] & [-3, -3] & [10, 10] & [-4, -4] & [8, 8] & [0, 0] & [0, 0] \\ [-4, -4] & [9, 9] & [4, 4] & [0, 0] & [-3, -3] & [10, 10] & [-4, -4] & [8, 8] & [0, 0] & [0, 0] \end{pmatrix}$$

5. Interval DGPS

Consider the swarm localization problem with m robots $\mathcal{R}_i$,
 $i \in \{0, \dots, m\}$.

We express the localization problem in a fixed point manner:

$$\begin{aligned}
 x_i(t) &= x_i(0) + \int_0^t v_i \cos \theta_i d\tau \\
 y_i(t) &= y_i(0) + \int_0^t v_i \sin \theta_i d\tau \\
 \theta_i(t) &= \theta_i(0) + \int_0^t \omega_i d\tau \\
 v_i(t) &= v_i(0) + \int_0^t \alpha_i d\tau \\
 a_{ij}(t) &= x_j(t) - x_i(t) \\
 b_{ij}(t) &= y_j(t) - y_i(t) \\
 a_{ij}(t) &= a_{ij}(0) + \int_0^t (v_j \cos \theta_j - v_i \cos \theta_i) d\tau \\
 b_{ij}(t) &= b_{ij}(0) + \int_0^t (v_j \sin \theta_j - v_i \sin \theta_i) d\tau \\
 \mathbf{A}(t) &= \mathbf{A}(t) \oplus \mathbf{I} \\
 \mathbf{A}(t) &= -\mathbf{A}(t)^T \\
 \mathbf{A}(t) &= \mathbf{A}(t)^{\otimes 2} \\
 \mathbf{B}(t) &= \mathbf{B}(t) \oplus \mathbf{I} \\
 \mathbf{B}(t) &= -\mathbf{B}(t)^T \\
 \mathbf{B}(t) &= \mathbf{B}(t)^{\otimes 2}
 \end{aligned}$$

If we define the vector

$$\mathbf{X} = (x_i(t), y_i(t), \theta_i(t), v_i(t), \mathbf{A}(t), \mathbf{B}(t))$$

the set of equations can be written under the form $\mathbf{X} = \mathbf{G}(\mathbf{X})$.

$$\begin{aligned}
 [x_i](t) &= [x_i](t) \cap \left([x_i](0) + \int_0^t [v_i] \cdot \cos[\theta_i] d\tau \right) \\
 [y_i](t) &= [y_i](t) \cap \left([y_i](0) + \int_0^t [v_i] \cdot \sin[\theta_i] d\tau \right) \\
 [\theta_i](t) &= [\theta_i](t) \cap \left([\theta_i](0) + \int_0^t [\omega_i] d\tau \right) \\
 [v_i](t) &= [v_i](t) \cap \left([v_i](0) + \int_0^t [\alpha_i] d\tau \right) \\
 \begin{pmatrix} [a_{ij}](t) \\ [x_i](t) \\ [x_j](t) \\ [b_{ij}](t) \\ [y_i](t) \\ [y_j](t) \end{pmatrix} &= \begin{pmatrix} [a_{ij}](t) \\ [x_i](t) \\ [x_j](t) \\ [b_{ij}](t) \\ [y_i](t) \\ [y_j](t) \end{pmatrix} \cap \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} [a_{ij}](t) \\ [x_i](t) \\ [x_j](t) \\ [b_{ij}](t) \\ [y_i](t) \\ [y_j](t) \end{pmatrix} \\
 [a_{ij}](t) &= [a_{ij}](t) \cap \left([a_{ij}](0) + \int_0^t ([v_j] \cdot \cos[\theta_j] - [v_i] \cdot \cos[\theta_i]) d\tau \right) \\
 [b_{ij}](t) &= [b_{ij}](t) \cap \left([b_{ij}](0) + \int_0^t ([v_j] \cdot \sin[\theta_j] - [v_i] \cdot \sin[\theta_i]) d\tau \right) \\
 [\mathbf{A}](t) &= [\mathbf{A}](t) \cap [\mathbf{A}](t) \oplus \mathbf{I} \\
 [\mathbf{A}](t) &= [\mathbf{A}](t) \cap -[\mathbf{A}](t)^T \\
 [\mathbf{A}](t) &= [\mathbf{A}](t) \cap [\mathbf{A}](t)^{\otimes 2} \\
 [\mathbf{B}](t) &= [\mathbf{B}](t) \cap [\mathbf{B}](t) \oplus \mathbf{I} \\
 [\mathbf{B}](t) &= [\mathbf{B}](t) \cap -[\mathbf{B}](t)^T \\
 [\mathbf{B}](t) &= [\mathbf{B}](t) \cap [\mathbf{B}](t)^{\otimes 2}
 \end{aligned}$$

For the initialization, we have to take into account the GPS and DGPS measurements:

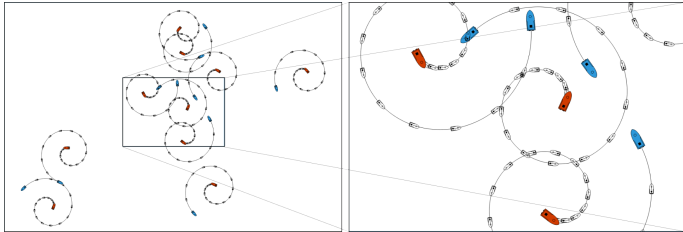
$$\begin{aligned}x_i(t_k) &\in [x_i](t_k) \\y_i(t_k) &\in [y_i](t_k) \\a_{ij}(t_k) &\in [a_{ij}](t_k) \\b_{ij}(t_k) &\in [b_{ij}](t_k)\end{aligned}$$

Test case

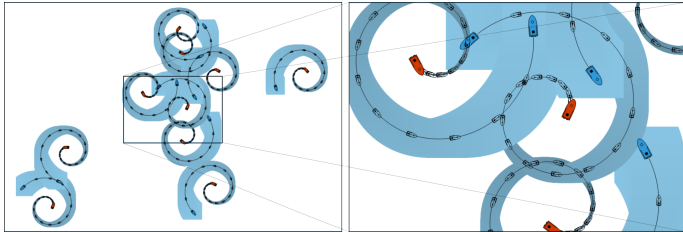
Consider $m = 10$ robots moving in the plane. The inputs are assumed to satisfy

$$\begin{aligned}\omega(t) &\in 1 - e^{-t} + [-0.01, 0.01] \\ \alpha(t) &\in 1 - e^{-t} + [-0.01, 0.01]\end{aligned}$$

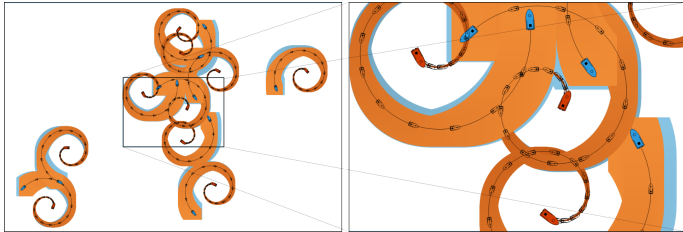
We took randomly a pose in the box $[-50, 50] \times [-50, 50]$ and $t_{\max} = 10\text{sec}$.



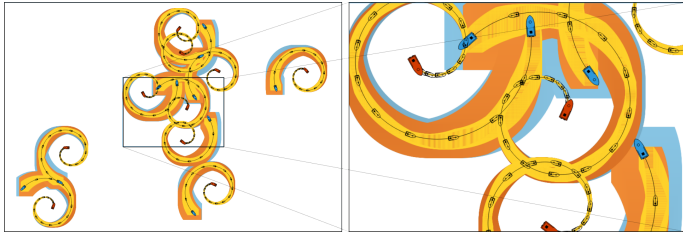
Trajectories of the 10 robots



Integration of the differential inclusion

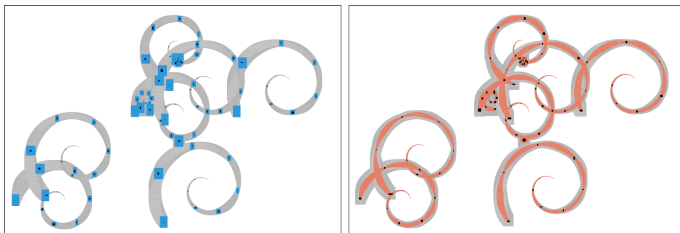


Combining the GPS with the dead reckoning, without the potential contractor



With the $(\max, +)$ potential contractor

Let us have a look of the interval uncertainties viewed by $\mathcal{R}_1$.
This view is indeed what can be used by $\mathcal{R}_1$ to avoid the collision
with other robots or to contribute to maintain a formation.



Viewed by $\mathcal{R}_1$: without (gray) and with (red) the potential contractor

References

- ❶ DGPS: Differential Global Positioning Systems [6]
- ❷ Source code: <https://www.ensta-bretagne.fr/jaulin/dgps.html>
- ❸ Video associated to the test-case
<https://youtu.be/xHPODUOYn0Y>
- ❹ Interval contractors: [4]. [9]
- ❺ Interval analysis: [12][1]
- ❻ Interval dynamical systems [8] [10] [13] [3]
- ❼ Max plus [11] [2][5][7]

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