Submeeting

Guaranteed state prediction of a group of underwater robots, using ellipsoids and zonotopes

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21 Juin 2022

The interest in groups of robots

Figure 1: Better surface covering

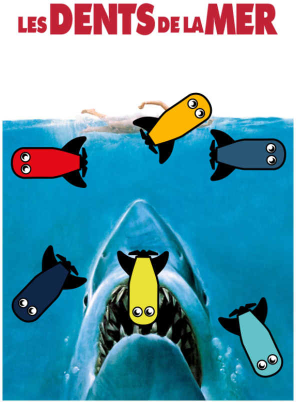


Figure 2: Better survivability

The development of groups of robots

Topic of research:

- localisation
- communication
- robustness
- fault tolerance
- obstacle avoidance
- modularity
- group behaviours

Challenges:

- many variables in the dynamical model of the group
- continuous and discrete variables
- environmental disturbances
- communication loss and delays
- actuators saturation’s

Guaranteed state propagation

Principle:

Initial state set $\mathcal{S}_0 \subset \mathbb{R}^n$
nonlinear mapping $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^n$
Enclosing set $\mathcal{S}_{\text{out}} \subset \mathbb{R}^n$ such that
$\boldsymbol{g}(\mathcal{S}_0) \subseteq \mathcal{S}_{\text{out}}$

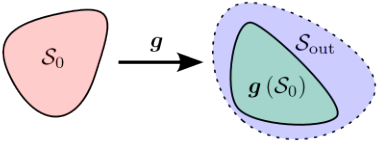


Figure 3: State enclosure

Interest:

- guaranteed state prediction algorithm
- can take part in mathematical proofs

Limitations:

- wrapping effect
- computational complexity

Common types of set:

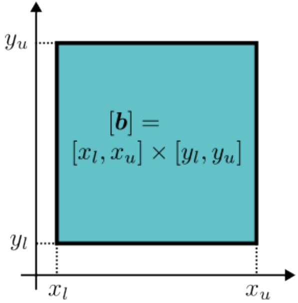
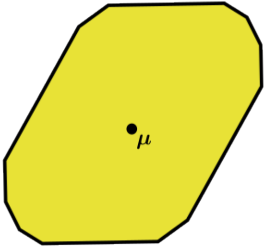
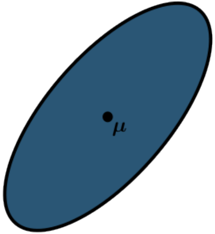


Figure 4: Boxes



$$\mathcal{Z}\left(\boldsymbol{\mu}, \boldsymbol{P}\right) = \left\{\boldsymbol{\mu} + \boldsymbol{P} \cdot \boldsymbol{e} | \boldsymbol{e} \in \mathbb{R}^k, \|\boldsymbol{e}\|_1 \leq 1\right\}$$

Figure 5: Zonotopes



$$\mathcal{E}\left(\boldsymbol{\mu}, \boldsymbol{\Gamma}\right) = \{\boldsymbol{x} \in \mathbb{R}^n | \\ \left(\boldsymbol{x} - \boldsymbol{\mu}\right)^T \boldsymbol{\Gamma}^{-T} \boldsymbol{\Gamma}^{-1} \left(\boldsymbol{x} - \boldsymbol{\mu}\right) \leq 1\}$$

Figure 6: Ellipsoids

Resemblance with Kalman filtering

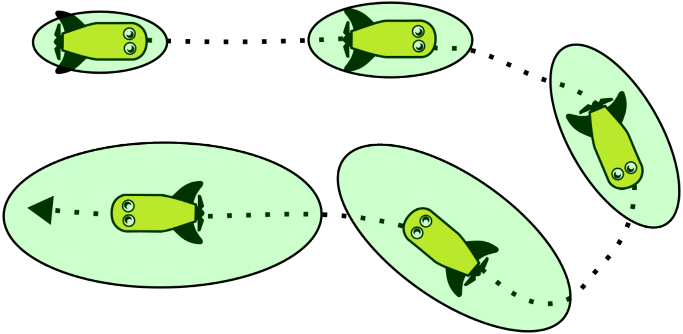


Figure 7: Propagation of the Kalman covariance ellipsoid

Types of systems

Discrete system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}_d(\boldsymbol{x}_k) \qquad (1)$$

Continuous system:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_c(\boldsymbol{x}) \qquad (2)$$

$$\boldsymbol{x}(t) = \phi(t, \boldsymbol{x}(0)) \qquad (3)$$

Hybrid system: a mix of continuous and discrete mappings

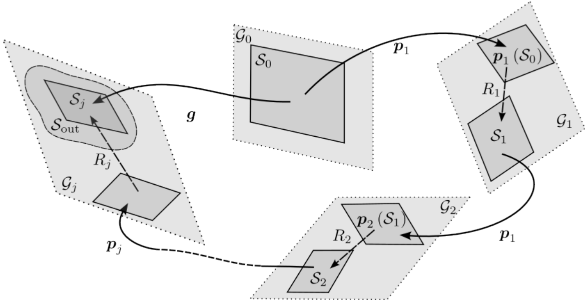


Figure 8: State propagation with hybrid system

$$\boldsymbol{g} = R_j \circ \boldsymbol{p}_j \circ \ldots \circ R_1 \circ \boldsymbol{p}_1 \qquad (4)$$

Computation of the propagation

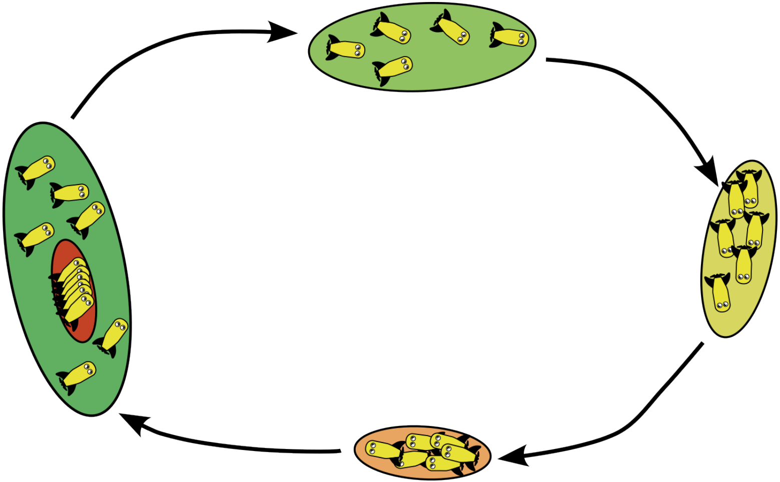


Figure 9: Illustration of the propagation

Consensus Example

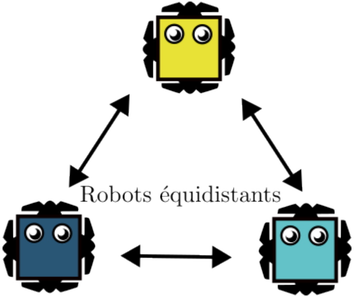


Figure 10: Formation control by consensus

three agent system with periodic pose measurement:

$$\dot{x}_i = v_i, \tag{5}$$
$$\dot{v}_i = u_i, \tag{6}$$
$$y_{i,k} = x_i(t_k), \tag{7}$$
$$t_k = k \cdot \delta_t \tag{8}$$

with $i \in \mathcal{I}_3$, $\mathcal{I}_3 = \{1, 2, 3\}$, $k \in \mathbb{N}$, $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$, $u_i \in \mathbb{R}$ and the measurement period $\delta_t > 0$

Consensus Example - Controller

The goal is to reach the consensus

$$\lim_{t\to\infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad (9)$$
$$\lim_{t\to\infty} \left\| v_i(t) - v_j(t) \right\| = 0 \quad (10)$$

for all $i, j \in \mathcal{I}_3$

Consensus protocol [Zheng et al.]:

$$\dot{u}_i(t) = \sum_{j\neq i} \left(y_{j,k} - x_i(t)\right) - c \cdot v_i(t),\ t \in (t_k, t_k + 1] \quad (11)$$

with the feedback gain $c > 0$.

Consensus Example - Conditioning

State Vector

$$\boldsymbol{z} = [z_i]_{i\in[1,9]}\,, \tag{12}$$
$$z_1(t) = x_2(t) - x_1(t), \tag{13}$$
$$z_2(t) = v_2(t) - v_1(t), \tag{14}$$
$$z_3(t) = x_3(t) - x_1(t), \tag{15}$$
$$z_4(t) = v_3(t) - v_1(t), \tag{16}$$
$$z_5(t) = x_3(t) - x_2(t), \tag{17}$$
$$z_6(t) = v_3(t) - v_2(t), \tag{18}$$
$$z_{7,k} = y_{2,k} - y_{1,k}, \tag{19}$$
$$z_{8,k} = y_{3,k} - y_{1,k}, \tag{20}$$
$$z_{9,k} = y_{3,k} - y_{2,k}. \tag{21}$$

Initial set:

$$\|\boldsymbol{z}(0)\| < e, \tag{22}$$
$$z_{6+j,0} = z_j(0), \text{ for } j \in [1:3] \tag{23}$$

with $e > 0$, equivalent to $\boldsymbol{z}(0) \in \mathcal{E}\left(0, \Gamma_0\right)$ with the matrix

$$\Gamma_0 = e \cdot \begin{bmatrix} \multicolumn{6}{c}{\boldsymbol{I}_6} & \boldsymbol{0}_{6,3} \\ 1 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & \boldsymbol{0}_{3,3} \\ 0 & 0 & 0 & 0 & 1 & 0 & \end{bmatrix} \tag{24}$$

Consensus Example - Propagation

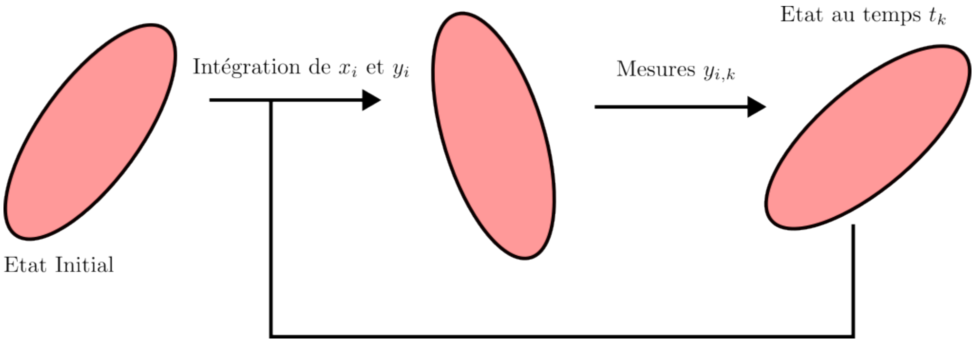


Figure 11: Propagation for consensus

Main libraries

CAPD-DynSys - "Computer Assisted Proofs in Dynamics" - Jagiellonian University

VNODE-LP - "A Validated Solver for Initial Value Problems in Ordinary Differential Equations" - McMaster University

Experiment of the week

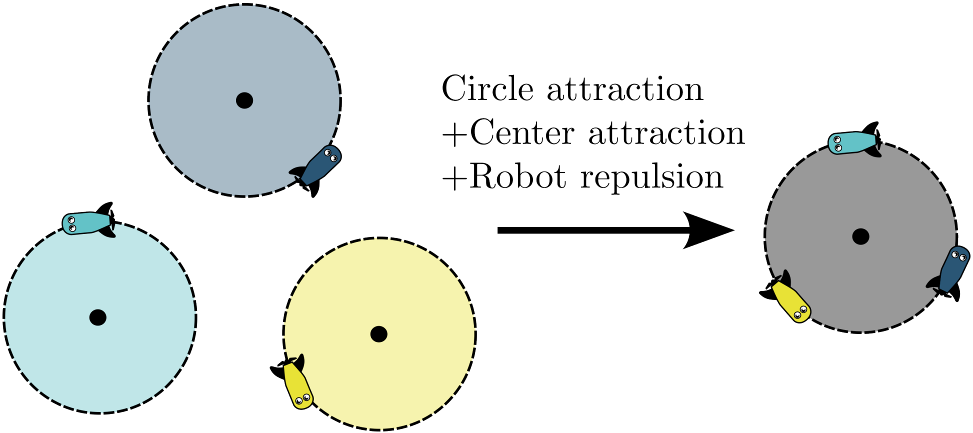


Figure 12: Circle consensus

Conclusion

Guarantedd state prediction:

- can be used for mathematical proofs in control theory
- gives a precise prediction for small sets
- must find a balance between pessimism and computation complexity

Current Limitations:

- libraries can be difficult to adapt to the robotic problems
- a good conditioning is required