A BRANCH AND PRUNE ALGORITHM FOR THE COMPUTATION OF GENERALIZED ASPECTS OF PARALLEL ROBOTS

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Mechanical manipulators, commonly called robots, are widely used in the industry to automatize various tasks. Robots are a mechanical assembly of rigid links connected by mobile joints. Some joints are actuated and they allow commanding the robot operating link, called its end-effector (or platform). One key characteristic of a robot is its reachable workspace, informally defined as the set of poses its end-effector can reach. Indeed, its size defines the scope of operational trajectories the robot will be able to perform. Robots comply with either a serial or a parallel (or possibly a hybrid) assembly, whether their links are connected in series or in parallel. Parallel robots [1, 2] present several advantages with respect to serial ones: They are naturally stiffer, leading to better accuracy with larger loads, and allow high speed motions. These advantages are contrasted by a more complicated design that yields difficulties for the computation and the analysis of their workspace. First, one pose of the robot's end-effector may be reached by several sets of actuated joint commands (which correspond to different working modes), and conversely one set of input commands may lead to several poses of its end-effector (which correspond to different assembly modes). Second, parallel robots generally have parallel singularities, i.e., specific configurations where they become uncontrollable and can even be damaged.

The kinematics of a parallel robot is modeled by a system of equations that relates the pose of its end-effector (which includes its position and possibly its orientation) to its commands. Hence computing the pose knowing the commands, or conversely, requires solving a system of equations, called respectively the direct and inverse kinematic problems. Usually, the number of pose coordinates, the number of commands and the number of equations are the same, and often referred to as the degrees of freedom (DOF) of the robot. Therefore, the relation between the pose and the command is generically a local bijection, which allows controlling the robot motion. However, in some non-generic configurations, the pose and the command are not anymore related by a local bijection. This may affect the robot behavior, e.g., destroying it if some commands are enforced with no corresponding pose. These non-generic cases are called robot singularities and they can be of two kinds [3]: Serial or parallel. One central issue in designing parallel robots is to compute connected sets of nonsingular configurations, so that the robot can safely operate inside those sets. Such a set is called a generalized aspect in [4] when it is maximal with respect to inclusion, i.e., generalized aspects are the connected components of the set of nonsingular configurations.

A key feature when computing the aspects is the certification of the results: Avoiding singularities is mandatory, and the connectivity between robot configurations must be certified. This ensures that the robot can actually move safely from one configuration to another. Few frameworks provide such certifications, among which algebraic computations and interval analysis. The cylindrical algebraic decomposition was used in [5], with the usual restrictions of algebraic methods and with a connectivity analysis limited to robots with two DOFs. Interval analysis was used in [6] for robots having a single solution to their inverse kinematic problem. Though limited, this method can still tackle important classes of robots like the Stewart platform. A quad-tree with certification of non-singularity was built in [7] for some planar robots with two DOFs. This method can be extended to higher dimensional robots, but it requires the a priori separation of working modes by adhoc inequalities, and is not certified with respect to connectivity.

In this work we propose a branch and prune algorithm incorporating the certification of the solutions and of their connectivity. This allows a fully automated and certified computation of what we call *connected sets of nonsingular configurations*, i.e., certified approximations of generalized aspects, from the model of arbitrary parallel robots, including robots with multiple solutions to their direct and inverse kinematic problems, without requiring any a priori study to separate their working modes. The algorithm is applicable to robots with an arbitrary number of DOF, although the complexity of the computations currently restricts its application to robots with three DOFs. It is also very flexible as it can naturally take into account additional constraints such as, e.g., arm collisions, obstacle avoidance or joint limits.

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