Exponential State Enclosures for the Verified Solution of Initial Value Problems for Ordinary Differential Equations

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Abstract

In recent years, interval methods have been studied widely to compute verified enclosures of the solutions of initial value problems for sets of ordinary differential equations with continuously differentiable right-hand sides [1]. However, uncertainty in initial conditions and parameters often leads to overestimation due to multiple dependencies on the same interval variables or due to the so-called wrapping effect. Both phenomena can make the computed state enclosures unnecessarily conservative and, hence, unusable for practical applications. For this reason, major research activities address techniques for a reduction of overestimation. These approaches preserve the property that the computed interval bounds contain the sets of reachable states with certainty.

Well-known techniques are high-order Taylor series expansions of the solution in time (combined with a suitable preconditioning of the state equations which are evaluated by means of interval arithmetic), and in the initial conditions (Taylor model representation of the solutions) as well as zonotope representations, or optimization-based approaches. Especially Taylor model representations and optimization-based procedures (possibly combined with consistency tests for a detection of overestimation) might become too time consuming if a real-time application of verified methods is desired. Such real-time applications are necessary if model-predictive control techniques are developed for uncertain continuous-time systems, where guaranteed enclosures of the predicted states have to be determined over a finite time horizon within predefined fixed sampling intervals.

For that reason, a computationally efficient enclosure technique has been developed recently which is based on a definition of the state enclosures in the form of exponential functions in time. For linear dynamic systems, these state enclosures can be determined by an iteration scheme which either exploits real-valued interval arithmetic (for systems with aperiodic dynamics) or complex-valued interval arithmetic (for oscillatory dynamics) [2, 3].

In this presentation, the prerequisites are discussed under which the above-mentioned exponential enclosure technique can be applied successfully to compute tight state enclosures by eliminating the wrapping effect. Practical application scenarios related to typical tasks in the robustness analysis of dynamic system models from control engineering conclude this contribution [4].

References

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