

A 4 foils hydrofoiler regulation solution

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Abstract

The abstract should be one paragraph. It should summarise the problem, approach, and significance of the results.

1 Introduction

The global (and old) interest for foils. reduce drag...

$$\begin{aligned} [0, T_{SCd}^T] &= Q_{SCd} \otimes [0, -D_{Obs}, 0, 0] \otimes Q_{SCd}^* \\ Q_{SCd} &= \frac{1}{2} \cdot \int_0^t (Q_{SCd} \otimes [0, \omega_{opérateur}^T]) dt \\ Q_{SCd}(0) &= 1_Q \end{aligned} \tag{1}$$

The case of passive stavle foils (V and J)

The case of machanical regulation (Moth)

Previous realisations (state of the art)

Novelty of our approach (4 regulated foils)

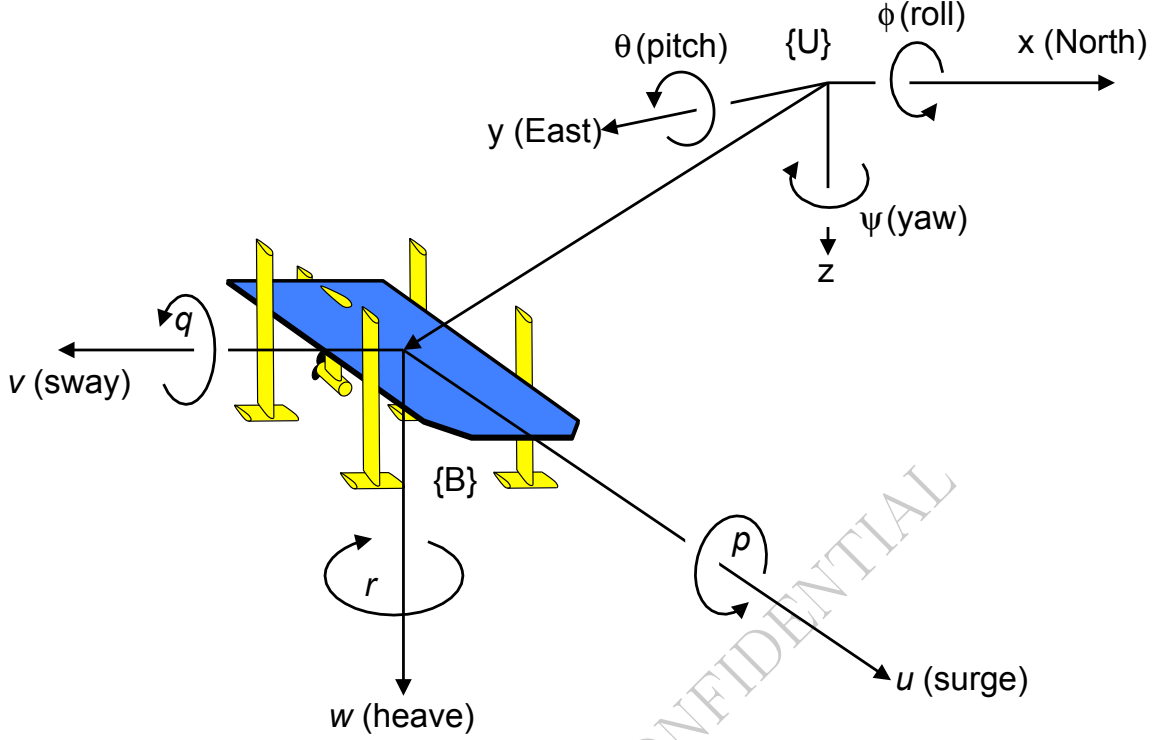


Figure 1: Notation and frames definition

2 System description

Description of the Crapeaud. Mechanics, Electronics, architectures, foil profile...

2.1 Notation

In the sequel, 'bold' notation is reserved for vectors and matrices, while scalars are written classically. Let $\{U\}$ be the universal coordinate frame and $\{B\}$ be the body frame. In the sequel, $\boldsymbol{\eta}$ expresses the system state in $\{U\}$, $\boldsymbol{\nu}$ is the system velocities in $\{B\}$, as stated on Equations (2) and illustrated at Figure 1.

$$\begin{aligned}
 \boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^T \\
 \boldsymbol{v} &= [u, v, w]^T \\
 \boldsymbol{\omega} &= [p, q, r]^T \\
 \boldsymbol{\nu} &= [\boldsymbol{v}^T, \boldsymbol{\omega}^T]^T
 \end{aligned} \tag{2}$$

The system kinematic model is classically taken as :

$$\dot{\boldsymbol{\eta}}_u = \mathbf{R}(\phi, \theta, \psi) \cdot \boldsymbol{\nu} \tag{3}$$

where $\mathbf{R}(\phi, \theta, \psi)$ denotes the kinematic relation between velocities expressed in the body frame $\{B\}$ and universal frame $\{U\}$, using Euler angle (ϕ, θ, ψ) . Introduce also matrix $\mathbf{R}_3$ as the 3×3 rotation matrix from $\{B\}$ to $\{U\}$ (first nine components of $\mathbf{R}(\phi, \theta, \psi)$).

$$\mathbf{R}(\phi, \theta, \psi) = \begin{bmatrix} \cos \psi \cos \theta & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi & 0 & 0 & 0 \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & 0 & 0 & 0 \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi & 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & 0 & 0 & 0 & \cos(\phi) & -\sin(\phi) \\ 0 & 0 & 0 & 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \\ 0 & 0 & 0 & 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \quad (4)$$

2.2 System description and dynamic modeling

A simplified dynamic model of the system is presented. Note that this model is used for control design and partial (in simulation) validation. A more complete model is presented in [REF THESE LOIC]. The proposition will be finally experimented, in order to show its performance in real conditions. The system is composed with 8 organs, as depicted at Figure 2.

- Hull : a simple floating foam board, denoted H. The origin of the $\{B\}$ frame is located at the geometric center of the hull, coinciding with its center of mass.
- Four articulated elements, called *foiling structures* in the sequel, each of them identically composed with a foil and a leg of length L_L ; called $L_{B,s}$, $L_{B,p}$, $L_{S,s}$ and $L_{S,p}$, where B stands for 'Bow', S for 'Stern', s for 'starboard' and p for 'portside'. These structures are symmetrically mounted on the hull with a pivot articulations, at a distance L_F to the foil, acting around v axis of $\{B\}$ with angles denoted $\Delta = [\delta_{B,s}, \delta_{B,p}, \delta_{S,s}, \delta_{S,p}]^T$. The coordinates of foiling structures' center of mass is denoted $\mathbf{X}_{B/S,s/p}^G$, w.r.t $\{B\}$ frame:

$$\begin{aligned} \mathbf{X}_{B,s}^G &= [x_L + z_L \cdot \sin \delta_{B,s}, y_L, z_L \cdot \cos \delta_{B,s}]^T \\ \mathbf{X}_{B,p}^G &= [x_L + z_L \cdot \sin \delta_{B,p}, -y_L, z_L \cdot \cos \delta_{B,p}]^T \\ \mathbf{X}_{S,s}^G &= [-x_L + z_L \cdot \sin \delta_{S,s}, y_L, z_L \cdot \cos \delta_{S,s}]^T \\ \mathbf{X}_{S,p}^G &= [-x_L + z_L \cdot \sin \delta_{S,p}, -y_L, z_L \cdot \cos \delta_{S,p}]^T \end{aligned} \quad (5)$$

where x_L, y_L and z_L are depicted at Figure 2..

- Thruster and its rigid leg of length L_T , called *thruster structure* and denoted T in the sequel, are perpendicularly mounted in the hull. Coordinates w.r.t $\{B\}$ of thruster structure's center of mass is denoted $\mathbf{X}_T^G$.

$$\mathbf{X}_T^G = [-x_T, 0, z_T]^T \quad (6)$$

- Rudder, of length L_R and denoted R, is mounted at stern on a pivot articulation acting around w axis of $\{B\}$ with an angle denoted δ_R . Coordinates w.r.t $\{B\}$ of the rudder center of mass is denoted $\mathbf{X}_R^G$.

$$\mathbf{X}_R^G = [-x_R, 0, z_R]^T \quad (7)$$

- Load, called *charge* and denoted as C in the sequel, which represents the equivalent inertial effect of other system unmodelled elements : electronic architecture, batteries, wires and actuators. It is mounted on the hull at point $\mathbf{X}_C$.

$$\mathbf{X}_C^G = [-x_C, 0, 0]^T \quad (8)$$

Each organs are modeled with the following characteristics, for organ $i = H, R, T, L_{B/S,p/s}$ and C, cf. Figure 2. Note that in the rest of the section, all vectorial expression are defined with respect to the body frame $\{B\}$.

- Let m_i denote the mass of each component, each center of mass is located at coordinates $\mathbf{X}_i^G(\Delta)$, w.r.t. $\{B\}$, where gravity force $\mathbf{G}_i(\eta)$ applies. The total mass is expressed with $M = \sum_i m_i$ and resulting gravity force vector with $\mathbf{G}(\eta)$, acting at $\mathbf{X}_G(\Delta)$.

$$\mathbf{X}_G = \frac{\sum_i (m_i \cdot \mathbf{X}_i^G)}{M}, \mathbf{G} = \mathbf{R}_3^{-1} \cdot [0, 0, M \cdot g]^T \quad (9)$$

where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration.

- Buoyancy (immersed volume) is denoted $b_i(\eta, \Delta)$, the associated buoyant force is $\mathbf{B}_i(\eta, \Delta)$ and the position of the buoyancy center, w.r.t the body frame $\{B\}$ is expressed as vector $\mathbf{X}_i^B(\eta, \Delta)$. The buoyancy center is located at the volumetric center of the immersed part of each organs. The total immersed volume is $B(\eta, \Delta) = \sum_i b_i$ and the resulting application point is $\mathbf{X}_B(\eta, \Delta)$

$$\mathbf{X}_B = \frac{\sum_i (b_i \cdot \mathbf{X}_i^B)}{B}, \mathbf{B} = \mathbf{R}_3^{-1} \cdot [0, 0, -B \cdot \rho \cdot g]^T \quad (10)$$

where $\rho = 1000 \text{ kg/m}^3$ is the water density.

- Drag force, denoted $\mathbf{D}_i(\eta, \nu, \Delta)$ is applied on each system component as

$$\mathbf{D}_i = -1/2 \cdot \rho \cdot \mathbf{S}_i \cdot \mathbf{C}_i^D \cdot \begin{bmatrix} u_i \cdot \text{sign}(u_i) & 0 & 0 \\ 0 & v_i \cdot \text{sign}(v_i) & 0 \\ 0 & 0 & w_i \cdot \text{sign}(w_i) \end{bmatrix} \cdot \mathbf{v}_i \quad (11)$$

where $\mathbf{C}_i^D = \begin{bmatrix} C_i^{D,u} & 0 & 0 \\ 0 & C_i^{D,v} & 0 \\ 0 & 0 & C_i^{D,w} \end{bmatrix}$ denotes the drag coefficient matrix of organ i , $\mathbf{S}_i = \begin{bmatrix} S_i^u(\eta) & 0 & 0 \\ 0 & S_i^v(\eta) & 0 \\ 0 & 0 & S_i^w(\eta) \end{bmatrix}$ is the matrix composed with the projected immersed surfaces of organ i , along the 3 body frame axis (Parameters' value are given at Table 3), and $\mathbf{v}_i = [u_i, v_i, w_i]^T$ denotes the velocity of the buoyancy center as, $\mathbf{v}_i = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{X}_i^B$. For the sake of simplicity, the application point is assumed to be the buoyancy center $\mathbf{X}_i^B$.

- Lift force, denoted $\mathbf{L}_i(\eta, \nu, \Delta)$ is only considered on the foils. Let $\boldsymbol{\alpha}(\eta, \nu, \Delta) = [\alpha_{B,s}, \alpha_{B,p}, \alpha_{S,s}, \alpha_{S,p}]^T$ denote the attack angles of the 4 foils. Hence the lift is computed with:

$$\mathbf{L}_i = -1/2 \cdot \rho \cdot S_i \cdot C_i^L(\alpha_i) \cdot (u_i^2 + w_i^2) \cdot \begin{bmatrix} \sin \delta_i \\ 0 \\ \cos \delta_i \end{bmatrix} \quad (12)$$

where S_i denotes the surface of foil i , and the attack angle is computed as :

$$\alpha_i = \delta_i + \arctan w_i / u_i \quad (13)$$

The lift characteristics of the foils $C_i^L(\alpha_i)$ are given at Figure XXX. For the sake of simplicity, the application point is assumed to be located at the volumetric center of the foils, denoted $\mathbf{X}_i^L$. Hence :

$$\begin{aligned} \mathbf{X}_{B,s}^L &= [x_L + L_F \cdot \sin \delta_{B,s}, y_L, L_F \cdot \cos \delta_{B,s}]^T \\ \mathbf{X}_{B,p}^L &= [x_L + L_F \cdot \sin \delta_{B,p}, -y_L, L_F \cdot \cos \delta_{B,p}]^T \\ \mathbf{X}_{S,s}^L &= [-x_L + L_F \cdot \sin \delta_{S,s}, y_L, L_F \cdot \cos \delta_{S,s}]^T \\ \mathbf{X}_{S,p}^L &= [-x_L + L_F \cdot \sin \delta_{S,p}, -y_L, L_F \cdot \cos \delta_{S,p}]^T \end{aligned} \quad (14)$$

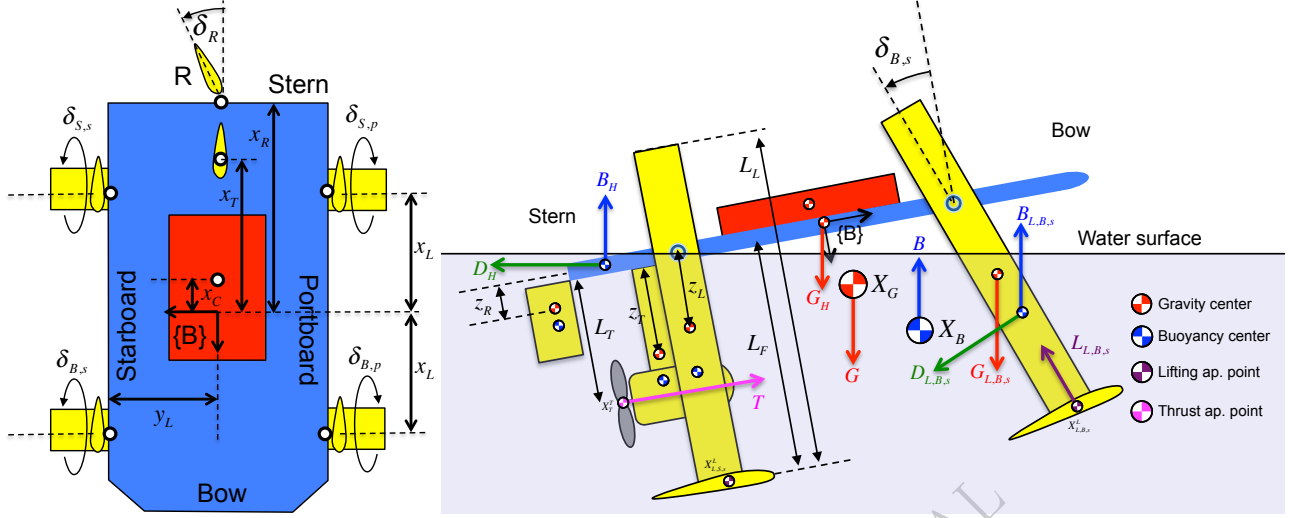


Figure 2: Forces applying on the system

- Thrust $\mathbf{T}$ induced by the thruster, and its input c_T , acts at point $\mathbf{X}_T^T$. The thruster's characteristic $\mathbf{T}(c_T)$ is given below.

$$\mathbf{X}_T^T = [-x_T, 0, L_T]^T \quad (15)$$

- The rudder induces a deflecting force $\mathbf{R}(\eta, \nu, \delta_R)$, function of the rudder angle δ_R , applied at the rudder's buoyancy center $\mathbf{X}_R^B$.

$$\mathbf{R} = -1/2 \cdot \rho \cdot S_R \cdot C_R^L(\delta_R) \cdot u_R^2 \cdot \begin{bmatrix} -\sin \delta_R \\ \cos \delta_R \\ 0 \end{bmatrix} \quad (16)$$

The rudder characteristic $C_R^L(\delta_R)$ is detailed below.

Hence, total resulting force is expressed as vector $\mathbf{F}$ as :

$$\begin{aligned} \mathbf{F} &= \mathbf{G} + \mathbf{B} + \mathbf{T} + \mathbf{R} + \sum_i (\mathbf{D}_i + \mathbf{L}_i) \\ &= \mathbf{F}_{\bar{L}} + \sum_i \mathbf{L}_i \end{aligned} \quad (17)$$

where $\mathbf{F}_{\bar{L}} = \mathbf{G} + \mathbf{B} + \mathbf{T} + \mathbf{R} + \sum_i \mathbf{D}_i$ denotes the forces acting of the system without lift action, and $\sum_i \mathbf{L}_i$ is the lift effect. These forces generate torque with respect to the body frame, resulting in vector $\mathbf{\Gamma}$, expressed as :

$$\begin{aligned} \mathbf{\Gamma} &= \mathbf{B} \times (\mathbf{X}_B - \mathbf{X}_G) + \mathbf{T} \times (\mathbf{X}_T^T - \mathbf{X}_G) + \mathbf{R} \times (\mathbf{X}_R^B - \mathbf{X}_G) \\ &\quad + \sum_i (\mathbf{D}_i \times (\mathbf{X}_i^B - \mathbf{X}_G)) + \sum_j (\mathbf{L}_j \times (\mathbf{X}_j^L - \mathbf{X}_G)) \\ &= \mathbf{\Gamma}_{\bar{L}} + \sum_j \mathbf{L}_j \times (\mathbf{X}_j^L - \mathbf{X}_G) \end{aligned} \quad (18)$$

for $i = H, R, T, C$ and $j = L_{B/S,p/s}$ and where $\mathbf{\Gamma}_{\bar{L}}$ denotes the torques acting on the system, without lift effect. Finally, system dynamic model is written with respect to the body frame $\{B\}$ as

$$\begin{cases} \mathbf{F} &= \mathbf{M} \cdot \dot{\mathbf{v}} \\ \mathbf{\Gamma} &= \mathbf{I} \cdot \dot{\boldsymbol{\omega}} \end{cases} \quad (19)$$

where $\mathbf{M}$ and $\mathbf{I}$ are the mass and inertia matrices (including added mass and inertia coefficients).

2.3 Modeling the actuation system

The actuation system is relating forces and torques generated by the 6 actuators (4 foils, 1 rudder and 1 thruster) with the resulting action on the controlled degrees of freedom, expressed in the body frame. We choose to decouple the control problem as follows :

- the 4 foiling structures regulates the system altitude (distance above water) to a desired value, roll and pitch angles to zero, constantly.
- the rudder regulates the yaw angle,
- the thruster acts along u axis of $\{B\}$ and is set to a constant reasonable regime.

In the sequel, we focus on the 'useful action' of the actuators. For the sake of simplicity their drag effect along u axis of $\{B\}$ is not considered in the actuation model. Since the originality of this work relies on the foiling structure control, we only present their related actuation model.

The concerned degrees of freedom are along the w , p and q axis of $\{B\}$. Hence foiling structure actuation system relates the lifting actions $\mathbf{L}_i$ with the resulting force and torques denoted F_w , Γ_p and Γ_q in $\{B\}$. Let F_w^L be the contribution of the lifting effect of the foils on the w axis of $\{B\}$:

$$\begin{aligned} F_w^L &= \sum_i ([0, 0, 1] \cdot \mathbf{L}_i) \\ &= \underbrace{[\cos \delta_{B,s}, \cos \delta_{B,p}, \cos \delta_{S,s}, \cos \delta_{S,p}]}_{\mathbf{C}_L^w} \cdot [F_{B,s}, F_{B,p}, F_{S,s}, F_{S,p}]^T = \mathbf{C}_L^w \cdot \mathbf{F}_L \end{aligned} \quad (20)$$

where

$$\begin{aligned} F_i &= -\|\mathbf{L}_i\| = -1/2 \cdot \rho \cdot S_i \cdot C_i^L(\alpha_i) \cdot (u_i^2 + w_i^2), \text{ with } i = B/S, s/p \text{ and} \\ \mathbf{F}_L &= [F_{B,s}, F_{B,p}, F_{S,s}, F_{S,p}]^T \end{aligned} \quad (21)$$

The action on roll, along p axis of $\{B\}$, results in the torque Γ_p^L , computed with

$$\begin{aligned} \Gamma_p^L &= \sum_i ([1, 0, 0] \cdot (\mathbf{L}_i \times (\mathbf{X}_G - \mathbf{X}_i^L))) \\ &= \underbrace{\begin{bmatrix} \cos \delta_{B,s} \cdot [0, 1, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{B,s}^L) \\ \cos \delta_{B,p} \cdot [0, 1, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{B,p}^L) \\ \cos \delta_{S,s} \cdot [0, 1, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{S,s}^L) \\ \cos \delta_{S,p} \cdot [0, 1, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{S,p}^L) \end{bmatrix}}_{\mathbf{C}_L^p} \cdot [F_{B,s}, F_{B,p}, F_{S,s}, F_{S,p}]^T = \mathbf{C}_L^p \cdot \mathbf{F}_L \end{aligned} \quad (22)$$

The resulting effect on pitch is expressed with Γ_q^L as

$$\begin{aligned}
\Gamma_q^L &= \sum_i ([0, 1, 0] \cdot (\mathbf{L}_i \times (\mathbf{X}_i^L - \mathbf{X}_G))) \\
&= \underbrace{\begin{bmatrix} \cos \delta_{B,s} \cdot [1, 0, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{B,s}^L) - \sin \delta_{B,s} \cdot [0, 0, 1] \cdot (\mathbf{X}_G - \mathbf{X}_{B,s}^L) \\ \cos \delta_{B,p} \cdot [1, 0, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{B,p}^L) - \sin \delta_{B,p} \cdot [0, 0, 1] \cdot (\mathbf{X}_G - \mathbf{X}_{B,p}^L) \\ \cos \delta_{B,p} \cdot [1, 0, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{B,p}^L) - \sin \delta_{B,p} \cdot [0, 0, 1] \cdot (\mathbf{X}_G - \mathbf{X}_{B,p}^L) \\ \cos \delta_{S,p} \cdot [1, 0, 0] \cdot (\mathbf{X}_G - \mathbf{X}_{S,p}^L) - \sin \delta_{S,p} \cdot [0, 0, 1] \cdot (\mathbf{X}_G - \mathbf{X}_{S,p}^L) \end{bmatrix}^T}_{\mathbf{C}_L^q} \cdot [F_{B,s}, F_{B,p}, F_{S,s}, F_{S,p}]^T = \mathbf{C}_L^q \cdot \mathbf{F}_L
\end{aligned} \tag{23}$$

where $\mathbf{F}_L = [F_{B,s}, F_{B,p}, F_{S,s}, F_{S,p}]^T$. Then, the foiling structure actuation system is modeled with the (3×4) matrix $\mathbf{C}_L$ as :

$$\mathbf{C}_L = \begin{bmatrix} \mathbf{C}_L^w \\ \mathbf{C}_L^p \\ \mathbf{C}_L^q \end{bmatrix}; \begin{bmatrix} F_w^L \\ \Gamma_p^L \\ \Gamma_q^L \end{bmatrix} = \mathbf{C}_L \cdot \mathbf{F}_L \tag{24}$$

Remark : the control design process requires the inversion of relation (41). The fact that $\mathbf{C}_L$ has more columns than lines illustrates the system redundancy, and implies that the inverse of $\mathbf{C}_L$ is not unique. This inversion process will be explicitly tackled at section ??? where system redundancy will be considered.

The thruster produces a force $F_T = \|\mathbf{T}\|$ along u axis of $\{B\}$ at a point located at the bottom of the thruster's leg. It thus impacts the q and u degrees of freedom through following relations :

$$\begin{bmatrix} \Gamma_q^T \\ F_u^T \end{bmatrix} = \begin{bmatrix} \mathbf{L}_T - [0, 0, 1] \cdot \mathbf{X}_G \\ 1 \end{bmatrix} \cdot F_T = \mathbf{C}_T \cdot F_T \tag{25}$$

where F_u^T and Γ_q^T denote the thruster actions along u and q axis of $\{B\}$.

At least, the rudder acts on the r and y axis of $\{B\}$ as:

$$\begin{bmatrix} F_v^R \\ \Gamma_r^R \end{bmatrix} = \begin{bmatrix} \cos \delta_R \\ l_R - [1, 0, 0] \cdot \mathbf{X}_G \end{bmatrix} \cdot F_R = \mathbf{C}_R \cdot F_R \tag{26}$$

where $F_R = \|R\|$.

3 Control Design

As introduced earlier, the control objectives are decoupled with the following strategy :

- the 4 foils act along the 3 D.O.F w, p, q , w.r.t $\{B\}$, in order to regulate a desired altitude above water z_d , and 2 desired roll and pitch angles ϕ_d and θ_d , respectively.
- the rudder acts on the yaw dynamics and is open loop controlled,
- the thruster is set to a constant regime and produces a force along u axis of $\{B\}$.

Consider the following expressions as desired closed-loop acceleration for the system :

$$\begin{aligned}\ddot{z}_c &= \ddot{z}_d + K_P^z \cdot (z_d - z) + K_I^z \cdot \int_{\tau=0}^t (z_d(\tau) - z(\tau))d\tau + K_D^z \cdot (\dot{z}_d - \dot{z}) \\ \ddot{\phi}_c &= \ddot{\phi}_d + K_P^\phi \cdot (\phi_d - \phi) + K_I^\phi \cdot \int_{\tau=0}^t (\phi_d(\tau) - \phi(\tau))d\tau + K_D^\phi \cdot (\dot{\phi}_d - \dot{\phi}) \\ \ddot{\theta}_c &= \ddot{\theta}_d + K_P^\theta \cdot (\theta_d - \theta) + K_I^\theta \cdot \int_{\tau=0}^t (\theta_d(\tau) - \theta(\tau))d\tau + K_D^\theta \cdot (\dot{\theta}_d - \dot{\theta})\end{aligned}\quad (27)$$

where $K_{P,I,D}^{z,\phi,\theta}$ are positive gains. Clearly, if the system tracks these desired acceleration, second order convergence of $z_d - z$, $\phi_d - \phi$ and $\theta_d - \theta$ is achieved. The expression of these references in the body frame is expressed as:

$$\begin{aligned}\dot{w}_c &= \frac{1}{\cos \theta \cdot \cos \phi} \cdot (\ddot{z}_c + \dot{u} \cdot \sin \theta - \dot{v} \cdot \cos \theta \cdot \sin \phi) \\ \dot{p}_c &= \dot{\phi}_c - \dot{q} \cdot \sin \phi \cdot \tan \theta - \dot{r} \cdot \cos \phi \cdot \tan \theta \\ \dot{q}_c &= \frac{1}{\cos \phi} \cdot (\ddot{\theta}_c + \dot{r} \cdot \sin \phi)\end{aligned}\quad (28)$$

where it is assumed that $|\phi| < \pi/2$ and $|\theta| < \pi/2$ (Euler representation singularity). We now apply a classic 'computed torque' approach. The dynamic model along the w, p and q D.O.F. in $\{B\}$ can be extracted from (17) and (18) as:

$$\begin{bmatrix} F_w \\ \Gamma_p \\ \Gamma_q \end{bmatrix} = \begin{bmatrix} F_L^w \\ \Gamma_L^p \\ \Gamma_L^q \end{bmatrix} + \mathbf{C}_L \cdot \mathbf{F}_L \quad (29)$$

where F_w and F_L^w denote the component of $\mathbf{F}$ and $\mathbf{F}_L$ along the w axis of $\{B\}$, Γ_p and Γ_L^p denote the component of $\mathbf{\Gamma}$ and $\mathbf{\Gamma}_L$ along the p axis of $\{B\}$, and Γ_q and Γ_L^q denote the component of $\mathbf{\Gamma}$ and $\mathbf{\Gamma}_L$ along the q axis of $\{B\}$. For the sake of simplicity of the control design, we assume that $\mathbf{M} = \text{diag}\{M_u, M_v, M_w\}$ and $\mathbf{I} = \text{diag}\{I_p, I_q, I_r\}$ are diagonal matrices. It is then direct to state that the desired acceleration references (28) are tracked by the system if :

$$\mathbf{C}_L \cdot \mathbf{F}_L = \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} \quad (30)$$

The computation of the foils action requires the inversion of previous relation. One can classically use the Moore-Penrose pseudo inverse $\mathbf{C}_L^+ = \mathbf{C}_L^T \cdot (\mathbf{C}_L \cdot \mathbf{C}_L^T)^{-1}$ as :

$$\mathbf{F}_L^c = \mathbf{C}_L^+ \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} \quad (31)$$

where $\mathbf{F}_L^c = [F_{L,B,s}^c, F_{L,B,p}^c, F_{L,S,s}^c, F_{L,S,p}^c]^T$ denotes the desired foil lift action in order for the system to track the desired acceleration (28). Next step consists in evaluation the 4 desired lifting coefficients $C_i^L(\alpha_i)$, for $i = B/S, s/p$, by inversion of the lift physics expression (21), as :

$$C_i^L(\alpha_i) = \frac{-2}{\rho} \cdot \frac{F_{L,i}^c}{u_i^2 + w_i^2} \quad (32)$$

assuming that $u > 0$, always, and for $i = \text{B/S,s/p}$. The desired foil angle is then extracted from (13) as :

$$\delta_i = (C_i^L)^{-1} \left(\frac{-2}{\rho} \cdot \frac{F_{L,i}^c}{u_i^2 + w_i^2} \right) - \arctan w_i/u_i \quad (33)$$

where $(C_i^L)^{-1}$ denote the inverse characteristic of the lift coefficient for the foil i , for $i = \text{B/S,s/p}$.

3.1 Guidance strategy design

The guidance system aims at shaping the error function in order to cope with system's capabilities and to shape a desired transitory regime. In our study, the strategy is threefold:

- Consider that foil efficiency increases with velocity. During take-off, when velocity u is small, a small error will easily push actuators to saturation. Assuming that the desired altitude above water is denoted h_d , the following expression implements this behavior.

$$\begin{aligned} z_d &= h_d \cdot \tanh(k_z \cdot u) \\ \tilde{z} &= z_d - z \end{aligned} \quad (34)$$

where $\tilde{z}$ stands for the altitude error and k_z is a positive gain shaping the stiffness of the tanh function.

- Regulate a transitory positive pitching during take-off. Here, we are inspired by the natural strategies of pitching positively the system to improve the take-off transitory regime, as kite-surfers do or motor trimming generally allows on rubber boats. As permanent regime is achieved, a null pitch angle is expected.

$$\begin{aligned} \theta_d &= \theta_{\max} \cdot \tanh(k_\theta \cdot \tilde{z}) \\ \tilde{\theta} &= \theta_d - \theta \end{aligned} \quad (35)$$

where $\tilde{\theta}$ denotes the pitching error, $\theta_{\max}$ is the maximum desired pitching angle and k_θ is a positive gain.

- Consider a 'banking turn' effect, as planes while changing heading, for passengers comfort. The desired banking angle is known (REF) and used as shown above.

$$\begin{aligned} \phi_d &= \frac{r \cdot v}{g} \\ \tilde{\phi} &= \phi_d - \phi \end{aligned} \quad (36)$$

where $\tilde{\phi}$ denotes the roll angle error.

Finally the foil control expression is written with:

$$\begin{aligned} \ddot{z}_c &= \ddot{z}_d + K_P^z \cdot \tilde{z} + K_I^z \cdot \int_0^t \tilde{z}(\tau) d\tau + K_D^z \cdot \dot{\tilde{z}} \\ \ddot{\phi}_c &= \ddot{\phi}_d + K_P^\phi \cdot \tilde{\phi} + K_I^\phi \cdot \int_0^t \tilde{\phi}(\tau) d\tau + K_D^\phi \cdot \dot{\tilde{\phi}} \\ \ddot{\theta}_c &= \ddot{\theta}_d + K_P^\theta \cdot \tilde{\theta} + K_I^\theta \cdot \int_0^t \tilde{\theta}(\tau) d\tau + K_D^\theta \cdot \dot{\tilde{\theta}} \end{aligned} \quad (37)$$

used in equations (28 - 33) to obtain the 4 desired foil angles δ_i .

3.2 Redundancy management

Since the actuation system is modeled with the matrix $\mathbf{C}_L$, which contains more lines than columns, the system has a potentially manageable redundancy. Indeed, 4 foils are used to regulate 3 D.O.F. Moreover, one can easily verify that

$$\text{rank}(\mathbf{C}_L) = 3 \quad (38)$$

since the choice of system parameters (cf. Table 3) and for any attainable values of foil angles δ_i . Hence, as done in [REF ACT], an element $\mathbf{N}_L$ of the kernel of $\mathbf{C}_L$ is identified and used as a projector in the null space of $\mathbf{C}_L$, as exposed in the sequel.

$$\mathbf{N}_L = \text{Ker}(\mathbf{C}_L) / \mathbf{C}_L \cdot \mathbf{N}_L \cdot r_m = 0 \quad (39)$$

where $\mathbf{N}_L$ is a (4×1) element of the null space of $\mathbf{C}_L$, and for any value of the scalar r_m . Hence, Equation (31) can be modify according to :

$$\mathbf{F}_L^c = \mathbf{C}_L^+ \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} + \mathbf{N}_L \cdot r_m \quad (40)$$

and Equation (32) and (33) allow to compute the corresponding foil angles δ_i . By application of Equation (41), previous choice results in the following action, expressed in the $\{B\}$ frame as:

$$\begin{bmatrix} F_w^L \\ \Gamma_p^L \\ \Gamma_q^L \end{bmatrix} = \mathbf{C}_L \cdot \mathbf{C}_L^+ \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} + \underbrace{\mathbf{C}_L \cdot \mathbf{N}_L \cdot r_m}_{=0} \quad (41)$$

which is identical to the previous desired control action. Hence, following the classic task function approach (REF) a specific value of r_m can be computed in order to respect an added criterion, without affecting the primary control objective (28). An example where a desired value is imposed on a foil is given below.

3.2.1 Imposing a desired value of a foil angle

Let's impose a desired constant value for the foil angle δ_i^* . The corresponding attack angle α_i^* is computed according to Equation (13) and (21) provides the induced lift force F_i^* . Let's now computed the corresponding r_m in order to produce desired lift force F_i^* on the i^{th} of $\mathbf{F}_L^c$, by inversion of relation (40) as :

$$r_m = (\mathbf{N}_L(i))^{-1} \cdot \left(F_i^* - \mathbf{C}_L(i, :) \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} \right) \quad (42)$$

where $\mathbf{N}_L(i)$ is the i^{th} component of the (1×4) matrix $\mathbf{N}_L$ and $\mathbf{C}_L(i, :)$ is the i^{th} line of $\mathbf{C}_L$.

Table 1: Control gains used in simulation

	K_P	K_I	K_D
z_c	100	500	10
ϕ_c	800	0	100
θ_c	800	100	200

Table 2: Parameters of the guidance functions

k_z	$\theta_{\max}$	k_θ
0.8	0.2	5

4 Simulation results

In order to illustrate the performance of the solution proposed, we performed simulations with the parameters reported at table 3, where it is assumed that the charge does not have any hydrodynamic effect. The foils have the Eppler 817 profile reported at Figure XXX.

In the following, we present simulations results with an identical scenario : for the first 10 seconds, the thruster produces a constant force $T = 30N$, then for $t = 10s$ to $t = 20s$ the rudder is used with $\delta_R = 0.3rad$, and for $t = 20s$ to $t = 30s$ the rudder is oriented to $\delta_R = -0.3rad$.

In order to show the advantage of using a foiling structure, we first perform a simulation of the system without foiling structures. State evolution are given at Figure 3-up. The system reaches a constant velocity $u_\infty = 1.63m/s$. One should notice that the hull is a simple flat floating foam with very poor hydrodynamic characteristic. The torque produced by the thrust is clearly visible and induces a positive static pitch angle $\theta_\infty = 0.054rad$. At least the system performed the trajectory depicted at Figure 3-down.

The second simulation shows the system behavior without guidance function, with a constant $z_d = -0.2m$, $\theta_d = 0$ and $\phi_d = 0$. The control gains have been chosen according to table 1. The results are shown at Figure 4.

The analysis of Figure 4 shows that the system is achieving regulation during the first 10 seconds, while the rudder is fixed to $\delta_r = 0$. The velocity reaches a velocity of $u = 7.1 m/s$, showing the efficiency of the foiling strategy. The system takes-off at $t = 4.13 s$, after an actuation saturation period as the Foils activity Figures indicates, 4-down-right. The first rudder action ($\delta_R = 0.3 rad$ at $t = 10 s$) induces a roll angle dynamic which produces a constant roll of $\phi = 7e^{-3}rad$, while the system maintains regulation. The second rudder change ($\delta_R = -0.3 rad$ at $t = 20 s$) induces a bigger effects which pushes actuation to saturation and a bad system response to this disturbance. Finally, the system trajectory is shown at Figure 4-down-left.

The third simulation includes the guidance functions of equations (34), (35) and (36), with the control gains of Table 1 and guidance parameters of Table 2. Results are shown at Figure 5.

The system follows the guidance references, depicted in red at Figure 5-up. As shown on the actuation activity Figure 5-down-right, actuators are pushed to saturation only at the beginning, while the forward velocity u is small. Clearly the system response is better as the guidance references are followed. At least system trajectory is shown at Figure 5-down-left and exhibits a longer trajectory as the system efficiency is improved. Notice also that the achieved curvature is also improved, as one could expected from the banking strategy.

The fourth simulation implements the redundancy management proposed at section XXX. We follow previous

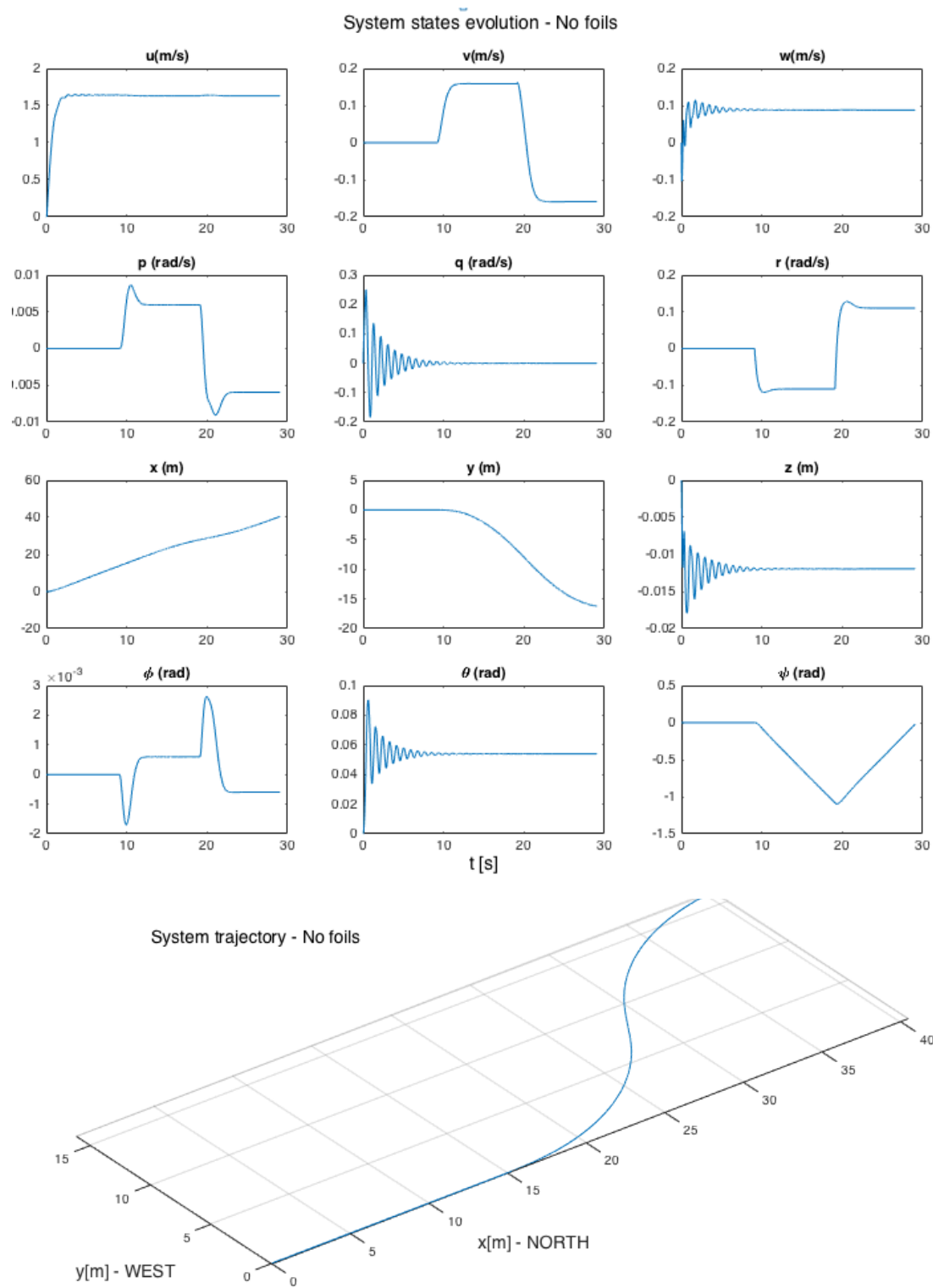


Figure 3: System states evolution (up) and trajectory (down), without foils

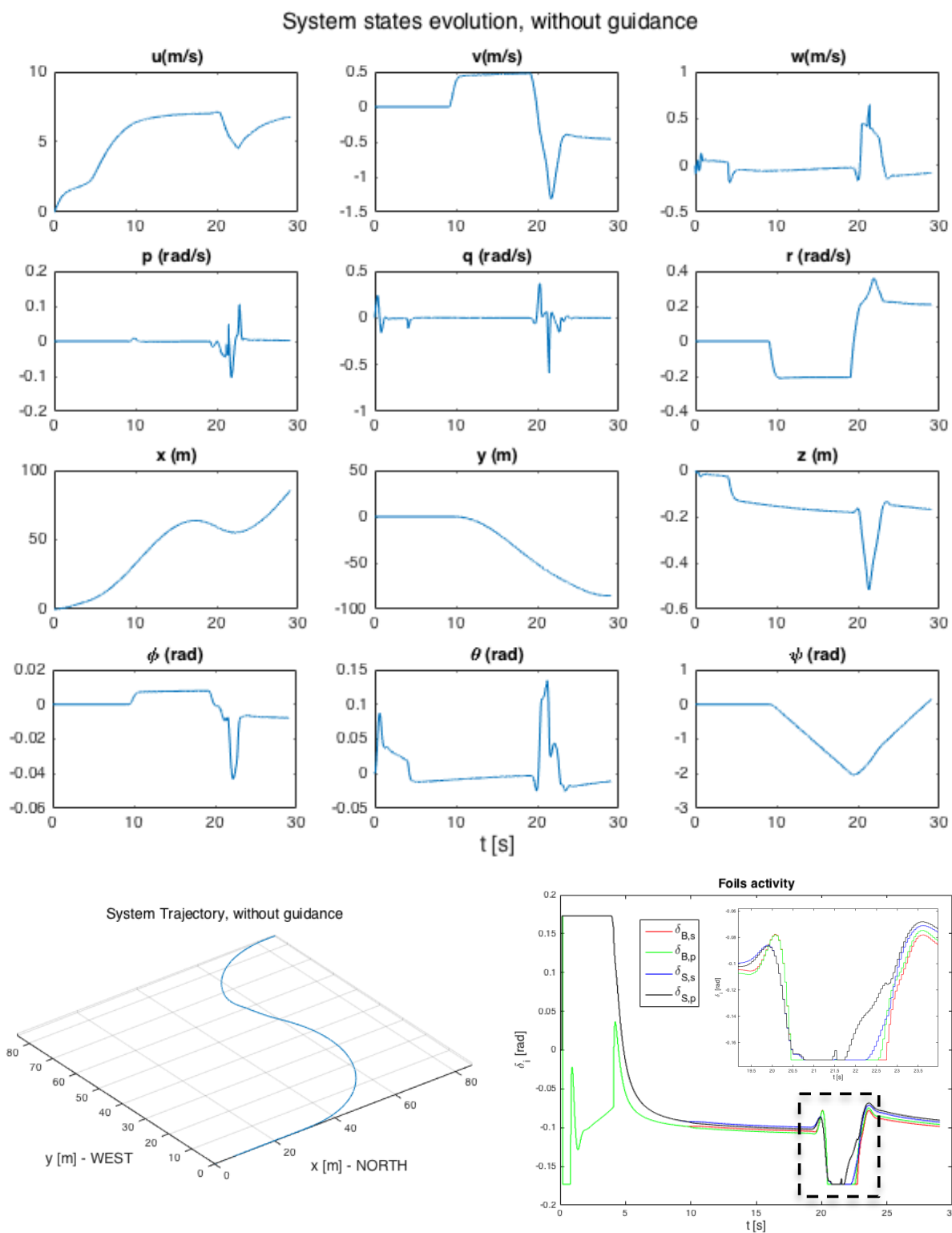


Figure 4: System states evolution (up), trajectory (down-left) and actuator activity (down-right), without guidance

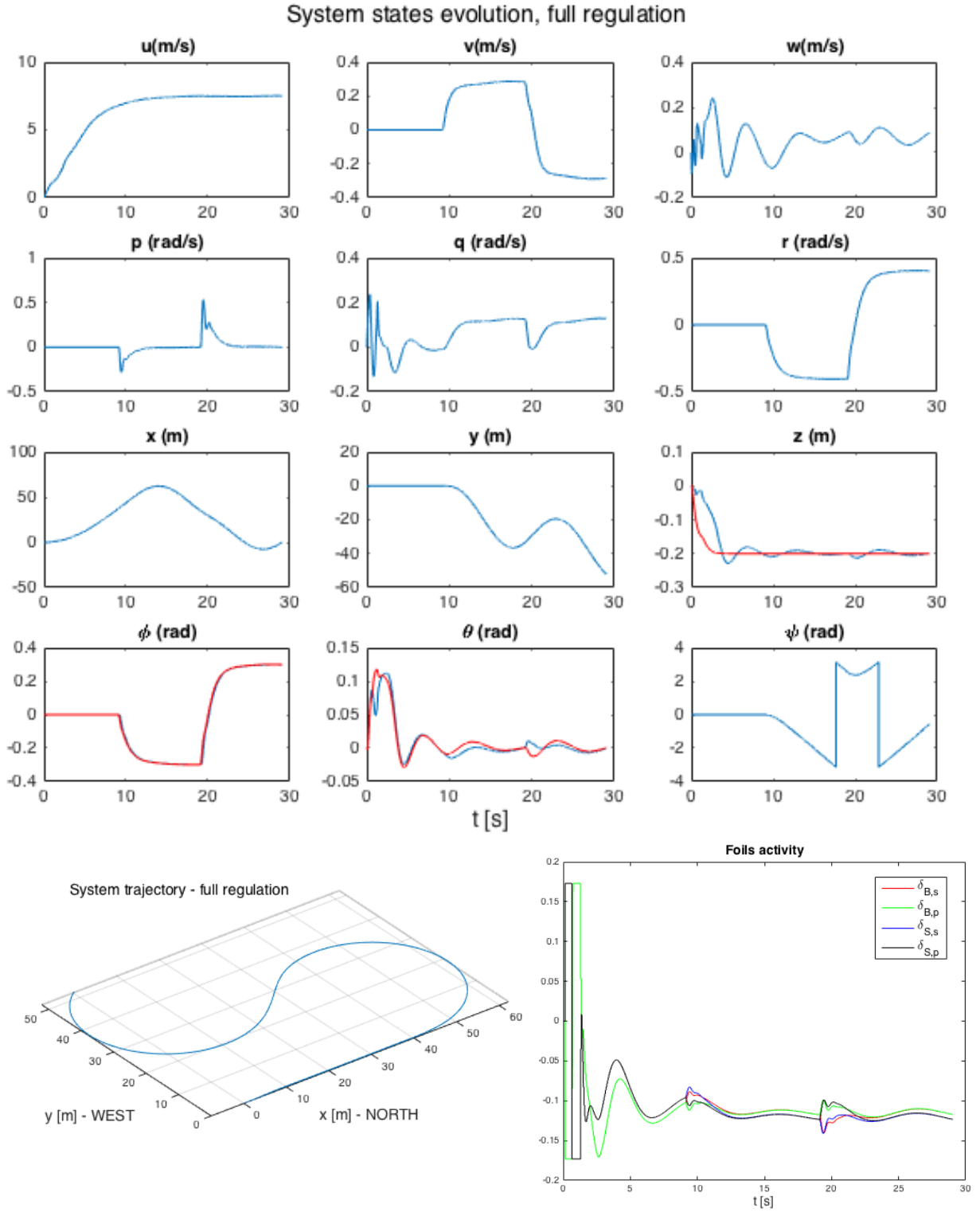


Figure 5: System states evolution (up), trajectory (down-left) and actuator activity (down-right), with guidance

Table 3: Systems parameters

	Pos / $\{B\}$ (m)	C_D (-)	Surf. Proj.(m ²)	mass (kg)	volume (m ³)
Hull	$[0, 0, 0]$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$	$\begin{bmatrix} 0.033 & 0 & 0 \\ 0 & 0.066 & 0 \\ 0 & 0 & 0.605 \end{bmatrix}$	0.9	0.0363
Foil's legs	$x_L = 0.3$ $y_L = 0.275$ $z_L = 0.1$ $L_L = 0.85$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.0085 & 0 & 0 \\ 0 & 0.085 & 0 \\ 0 & 0 & 0.0008 \end{bmatrix}$	0.7	$6.6e^{-4}$
Foils	$L_F = 0.525$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.0036 & 0 & 0 \\ 0 & 0.0008 & 0 \\ 0 & 0 & 0.036 \end{bmatrix}$	0.1	$2.5e^{-4}$
Thruster's leg	$x_T = -0.2$ $z_T = 0.12$ $L_T = 0.24$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.0024 & 0 & 0 \\ 0 & 0.048 & 0 \\ 0 & 0 & 0.021 \end{bmatrix}$	0.8	$6.6e^{-4}$
Thruster	-	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.0038 & 0 & 0 \\ 0 & 0.0252 & 0 \\ 0 & 0 & 0.0252 \end{bmatrix}$	3	0.0014
Rudder	$x_R = -0.6$ $z_R = 0.2$ $L_R = 0.4$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.0002 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix}$	1	$3.1e^{-5}$
Charge	$[0, 0, 0]$	-	-	10	-

scenario with a null r_m up to $t = 6s$, where a desired $\delta_{B,p}^* = -0.1$ rad is imposed on bow-portside foil . Then Equation (42) is implemented and used as in (40). Results are shown at Figure 6

Analyzing the results of Figure 6, it is clear that, as expected, the constraint $\delta_{B,p}^* = -0.1$ rad does not affect the system response. Nevertheless, one should note that the constraint has been chosen close to the regulation regime of previous simulation. Indeed, this type of constraint can easily pushes the other actuators to saturation, hence degrading the regulation capabilities.

5 Experimentations

We performed experimental validation on the system... To be included when experimentation will be completed

6 Conclusion and future work

What we did : regulate a 4-foiler system, include guidance law for take off and curve-banking. What remains to be done : robustness of the controller with a moving mass onboard...

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References

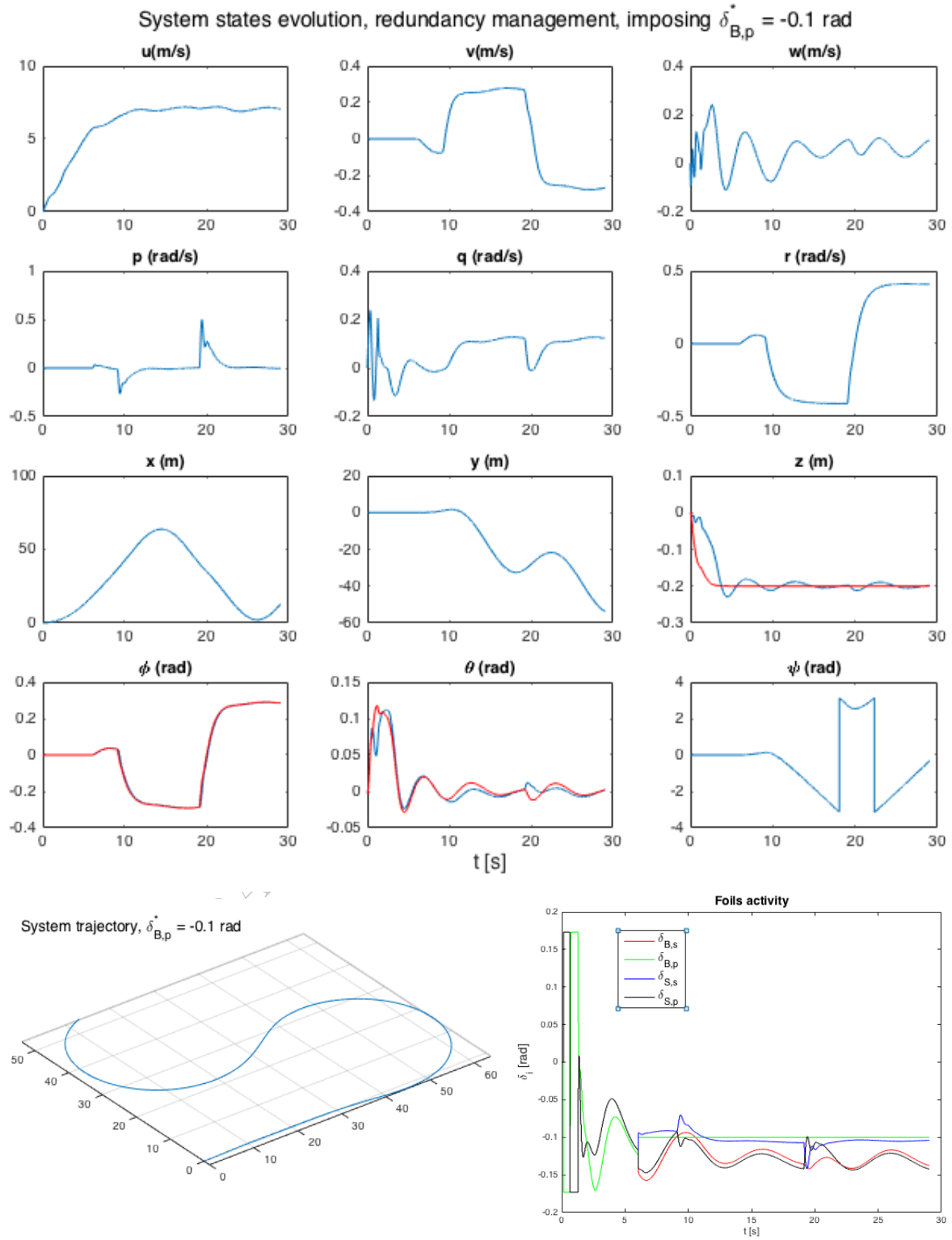


Figure 6: System states evolution (up), trajectory (down-left) and actuator activity (down-right), imposing $\delta_{B,p}^* = -0.1$ rad