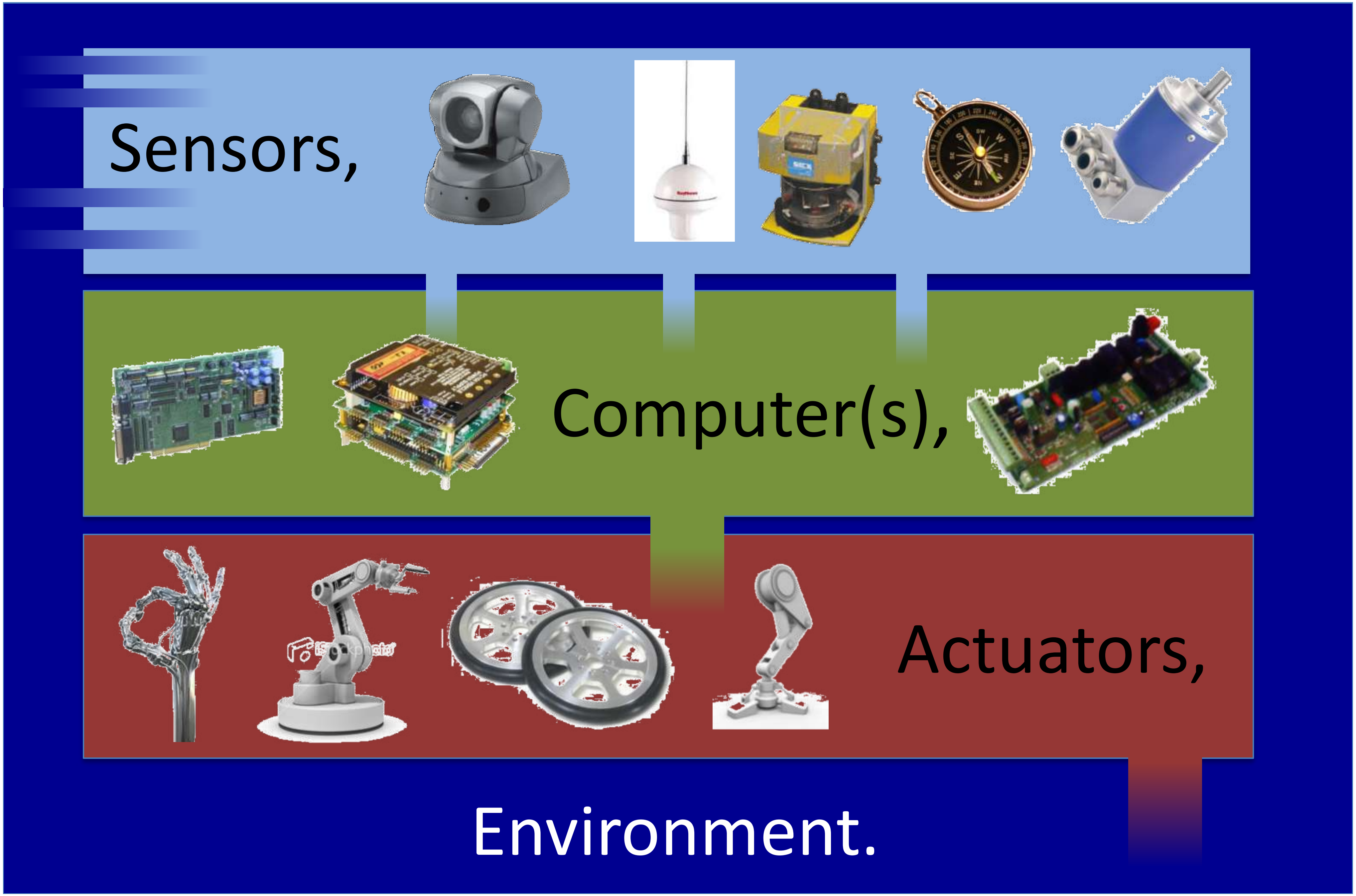
NGC Control Stucture
VT Path-following
Actuation redundancy

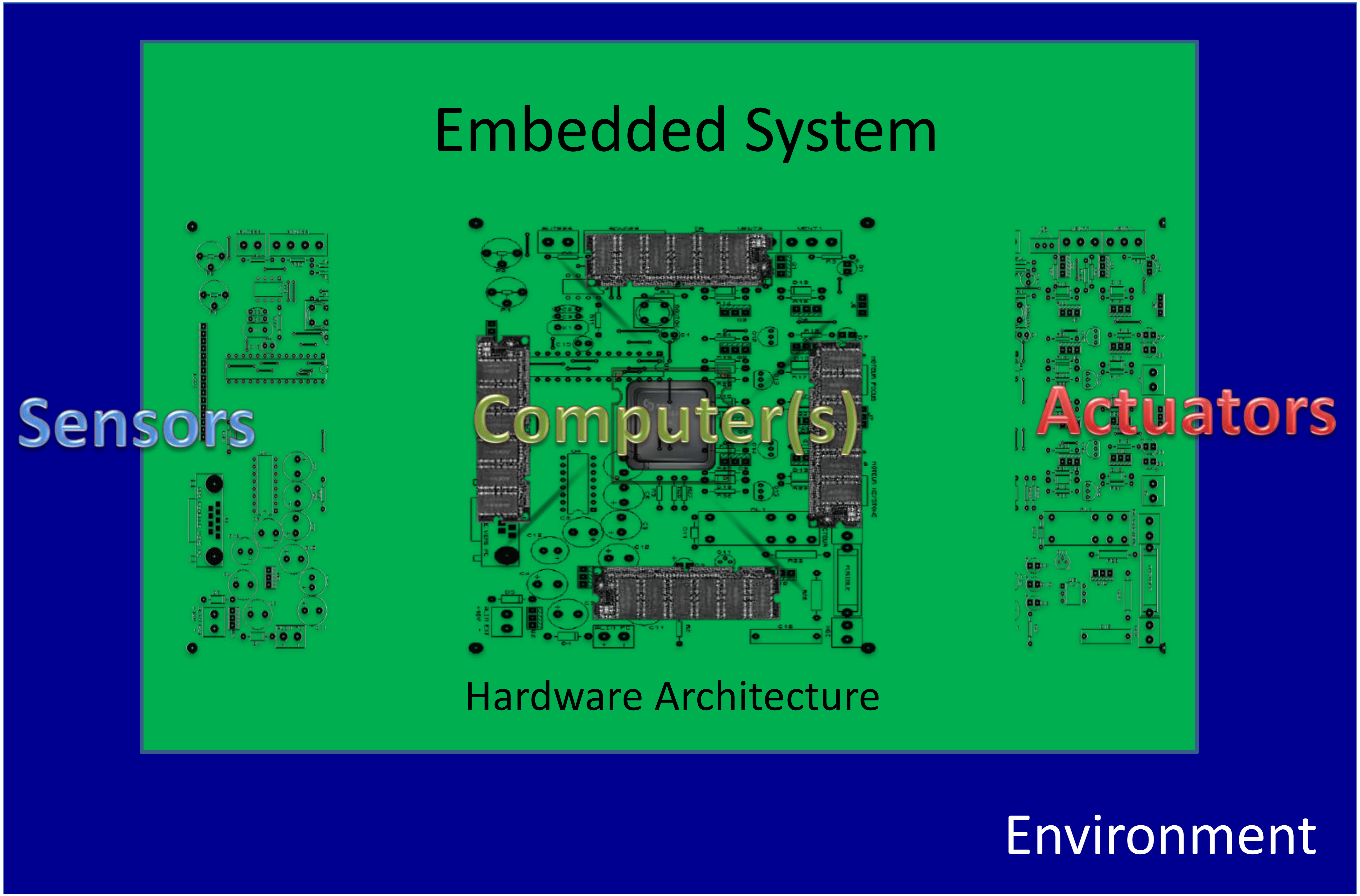
Lionel Lapierre, 4th Marine Robotics School, Goa, 11/02/2018

I. Anatomy

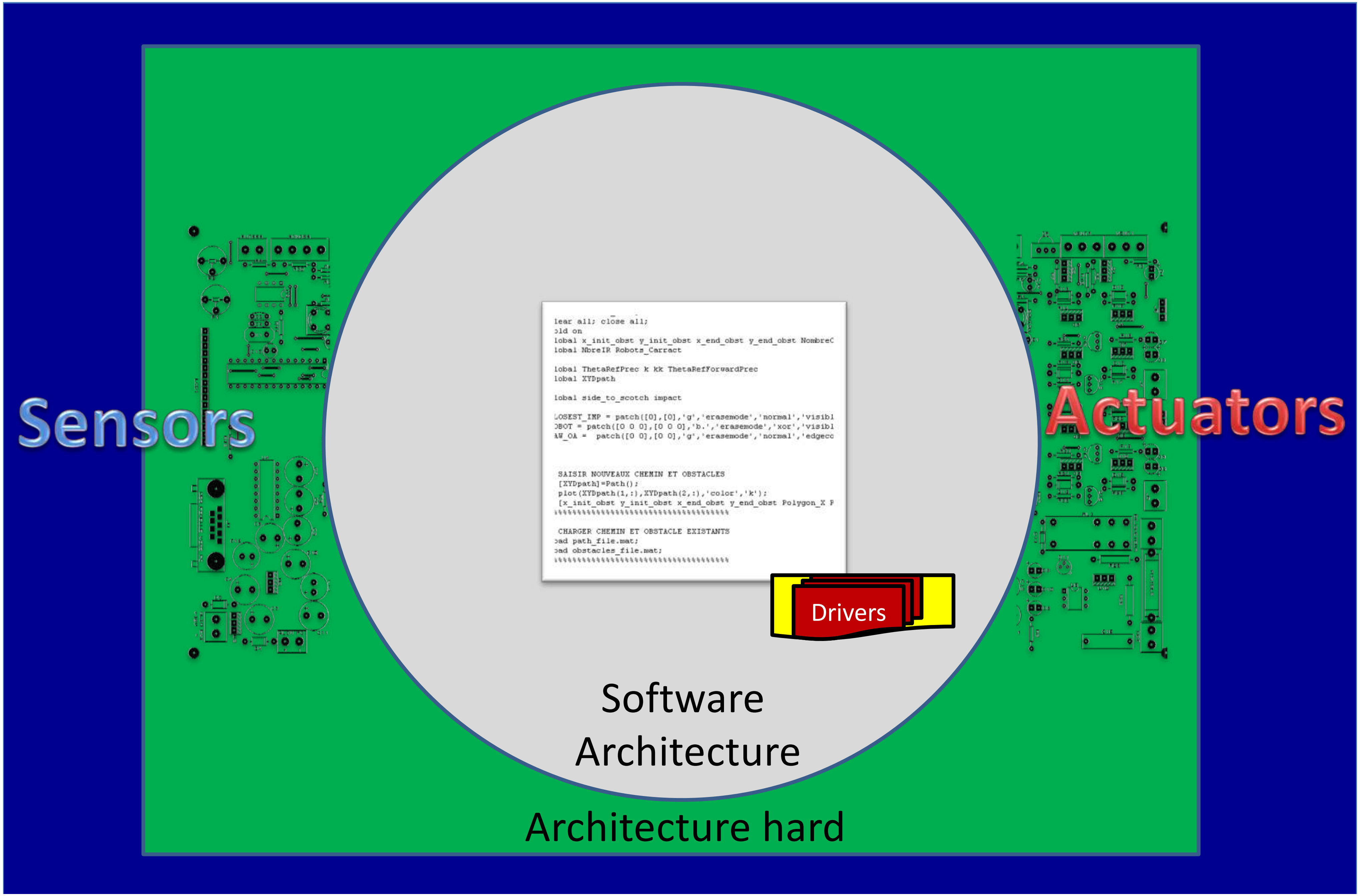
Robot components are:



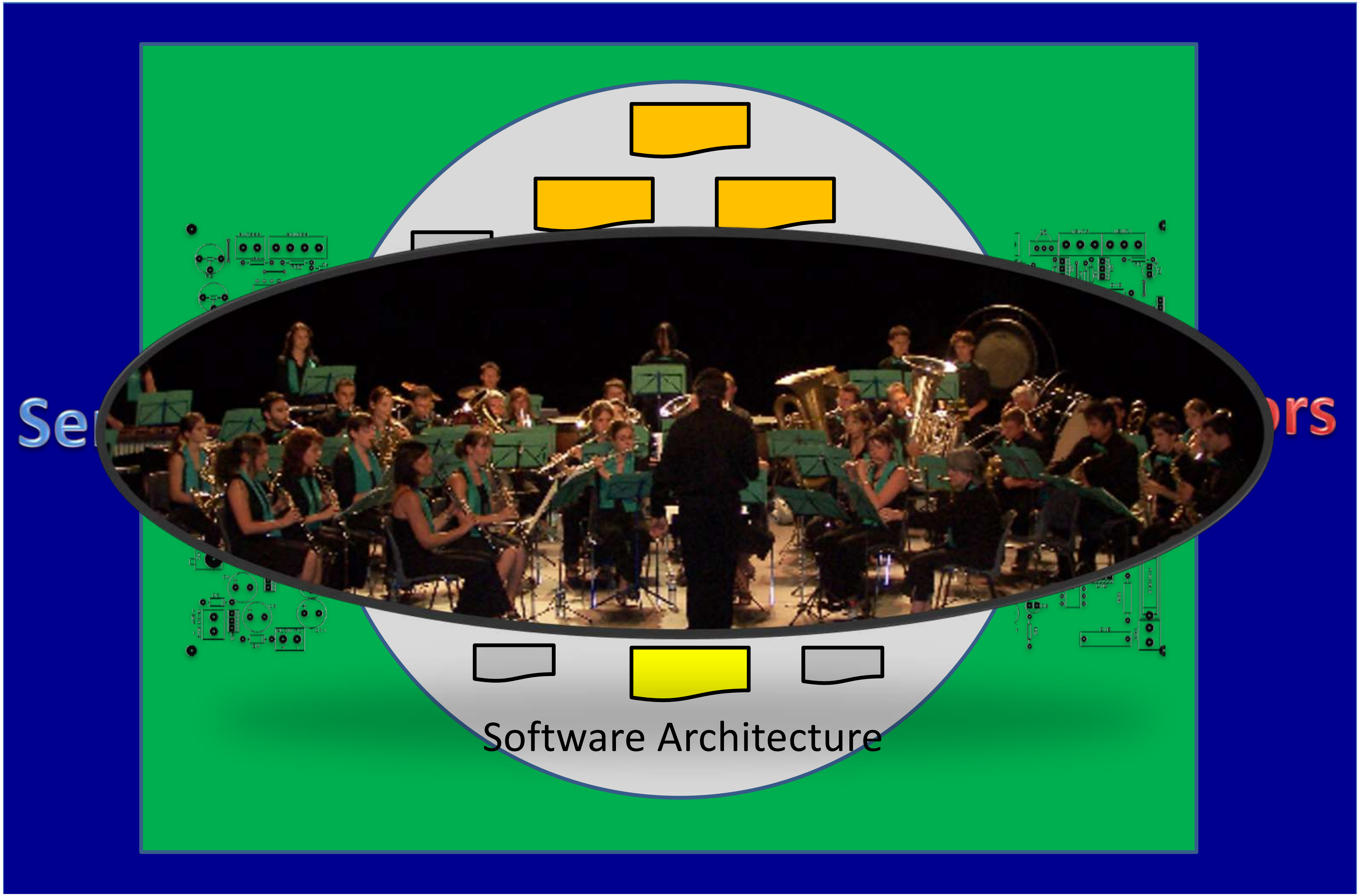
Electronics



Software Architecture



Software Architecture



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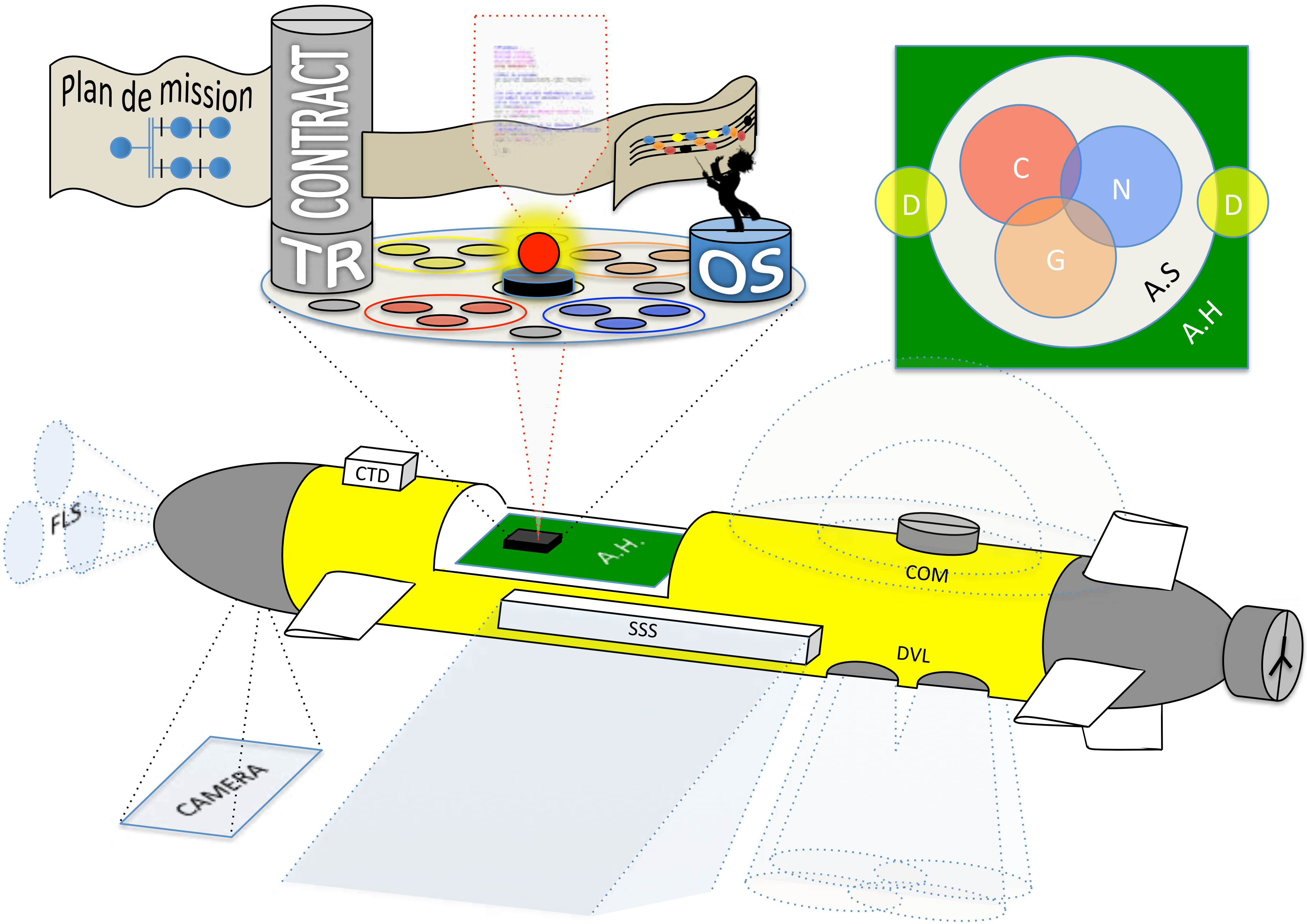
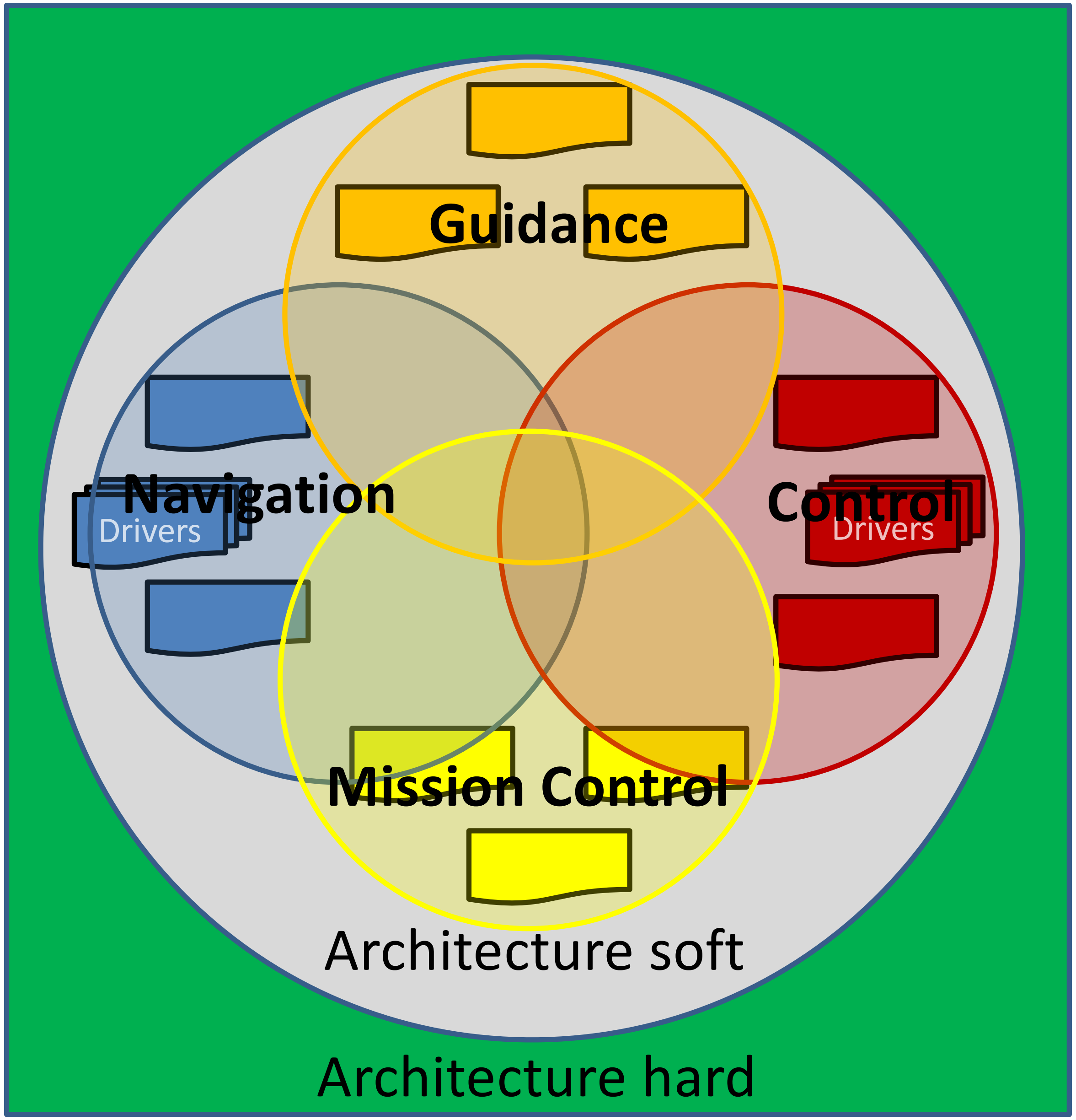


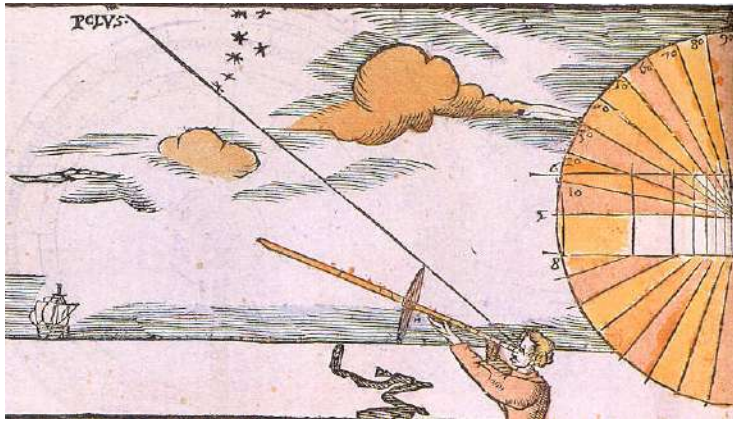
II. NGC Control Structure

The Functions
The Robot



Navigation

Where am I ?



Where I am going to ?

Navigation

Where am I?
Where am I going to?

The **Navigation System** provides estimates of the vehicle states **based on a set of motion sensor suites**.

Sensors data

↓

NAVIGATION

↓

Estimation of position, velocity

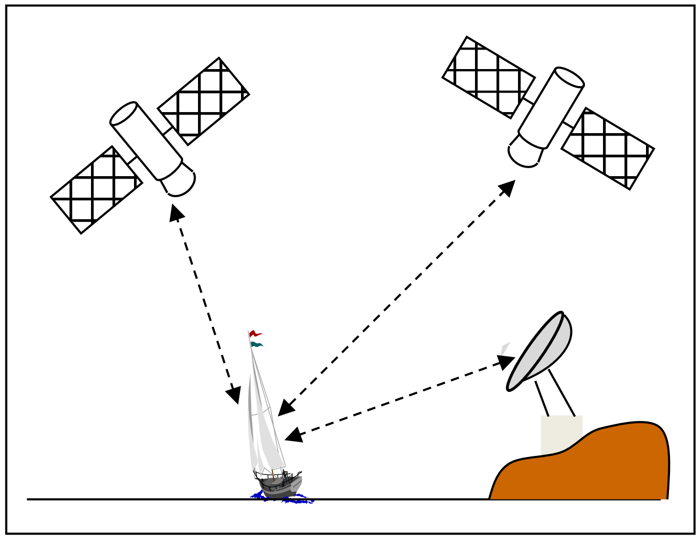
Navigation

Where am I?

Where am I going to?

The **Navigation System** provides estimates of the vehicle states **based on a set of motion sensor suites**.

Global



Navigation

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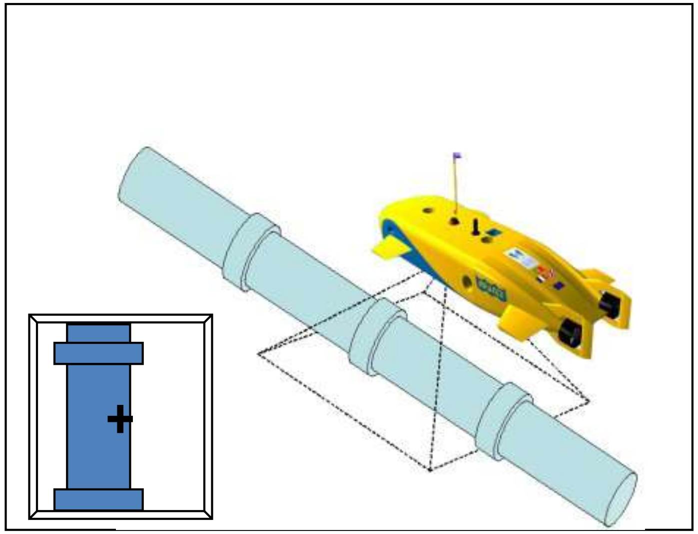
Local



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Where am I going to?

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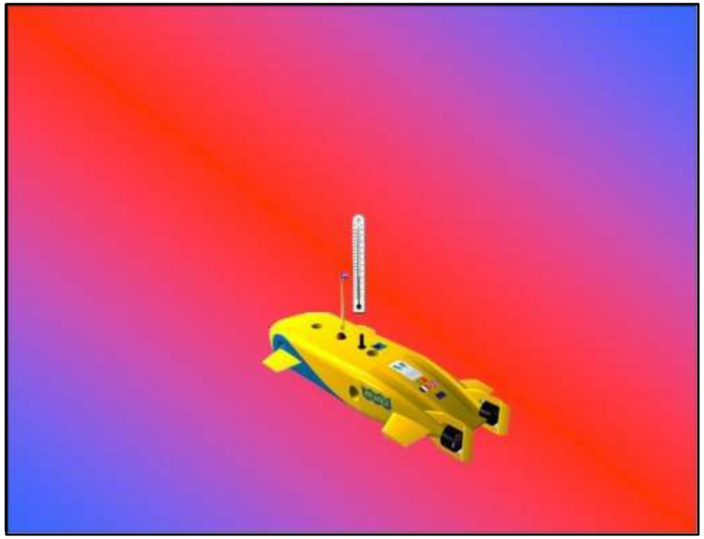
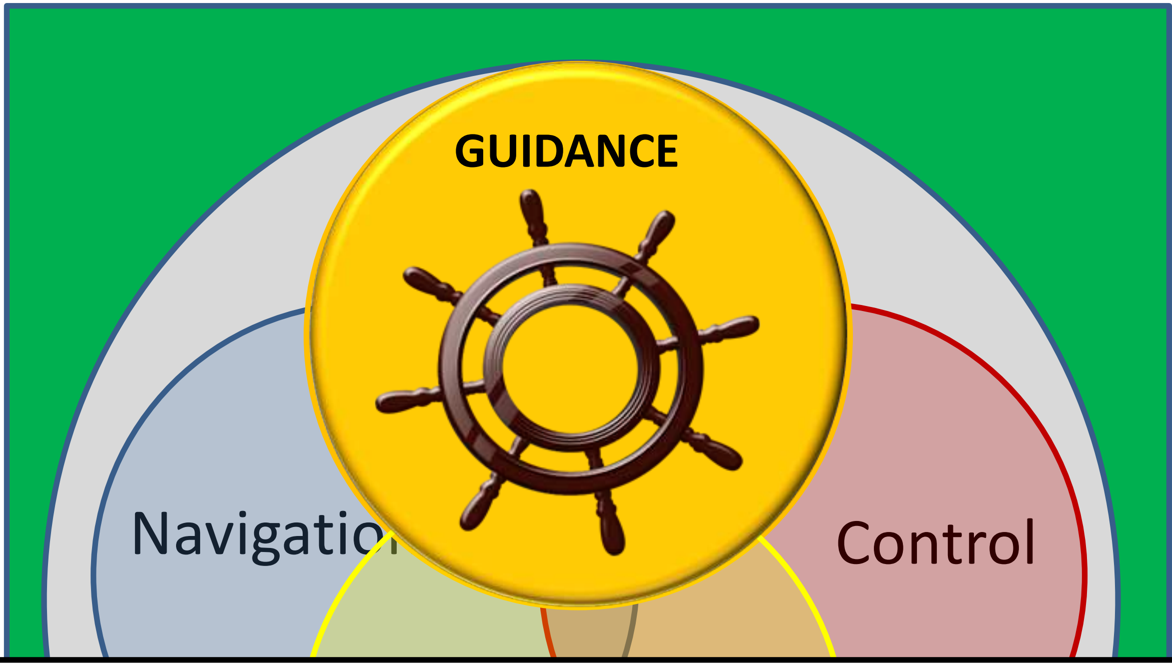
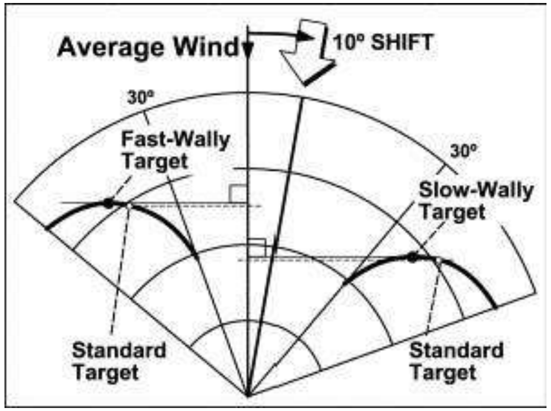


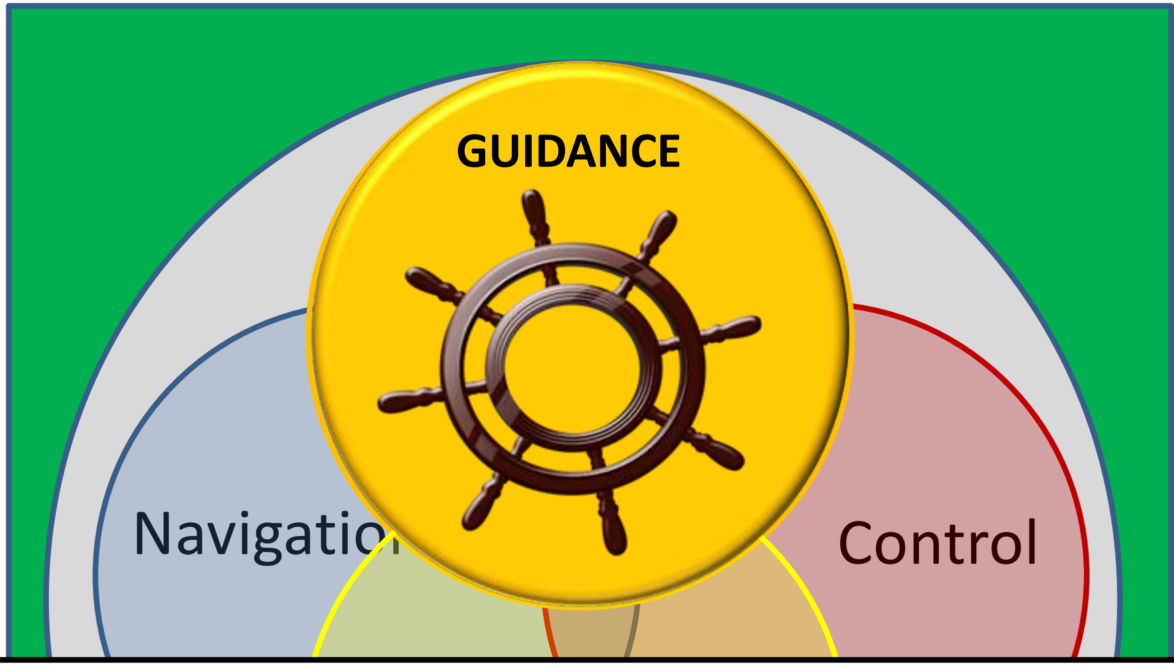
Guidance



How approaching the objective?

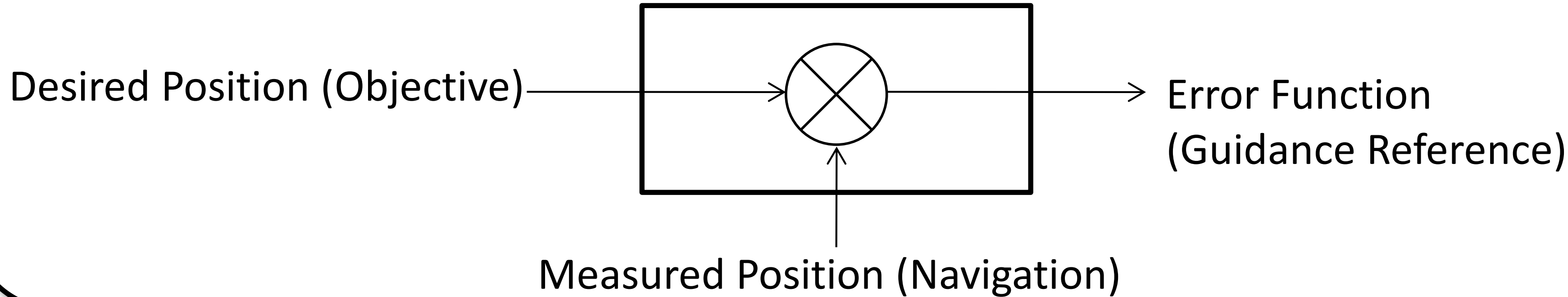


Guidance

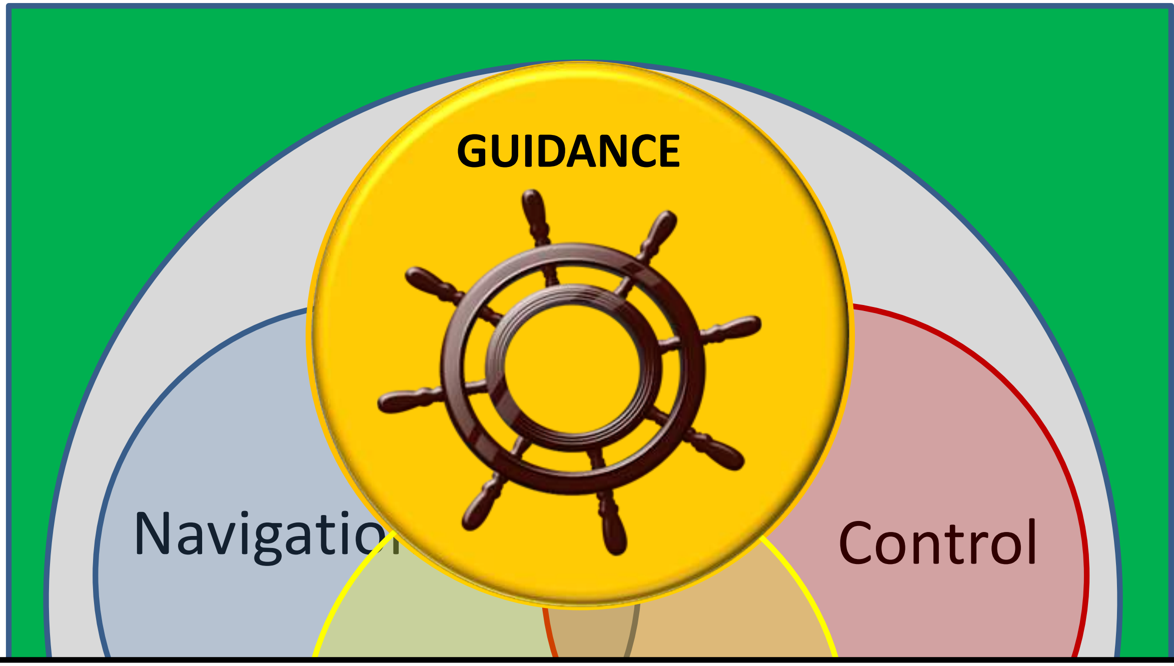


How approaching the objective?

The **Guidance system** processes Navigation/Inertial reference trajectory data and output **set-points for desired vehicle's velocity and attitude**



Guidance



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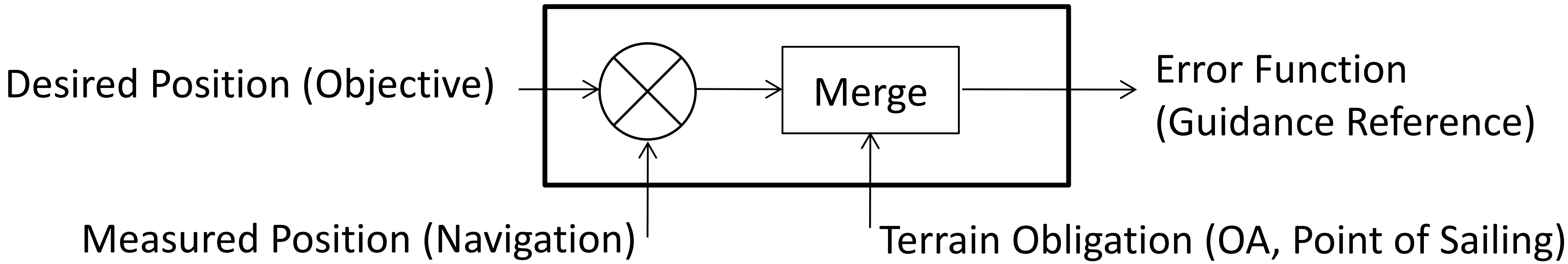
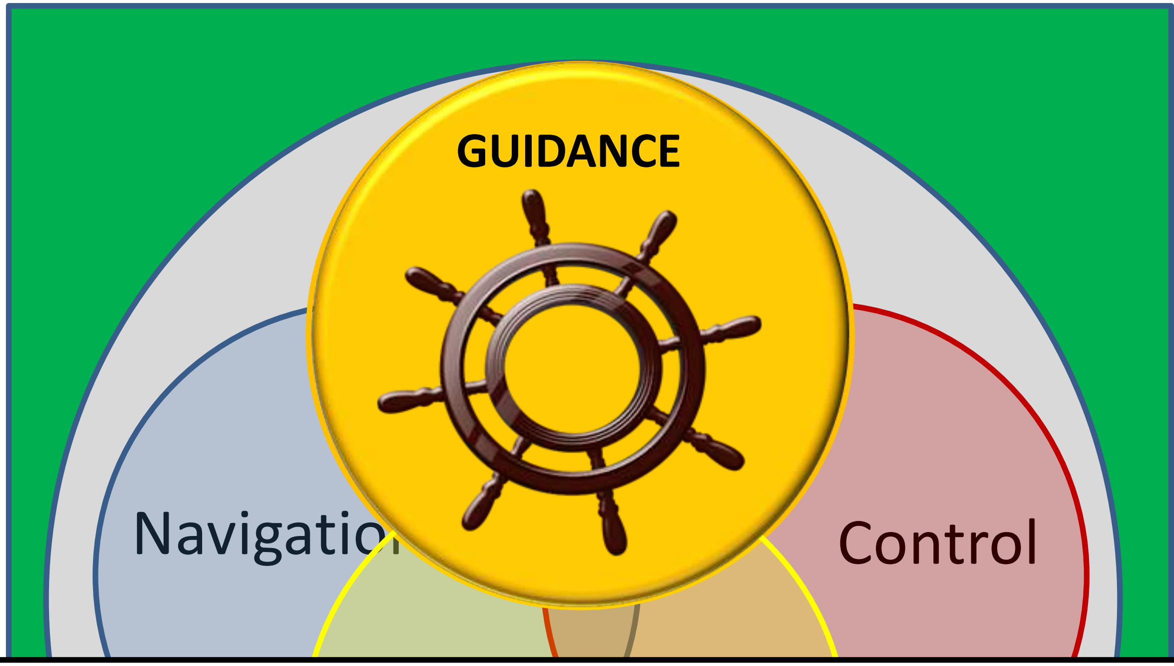


Guidance



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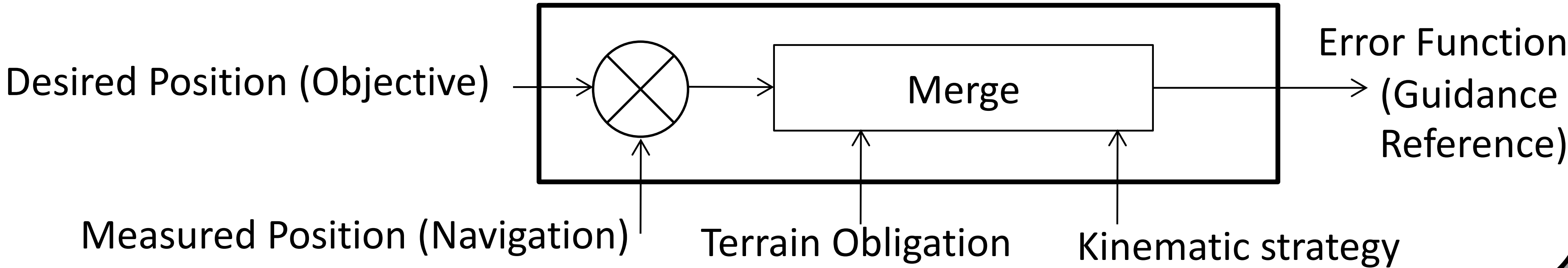
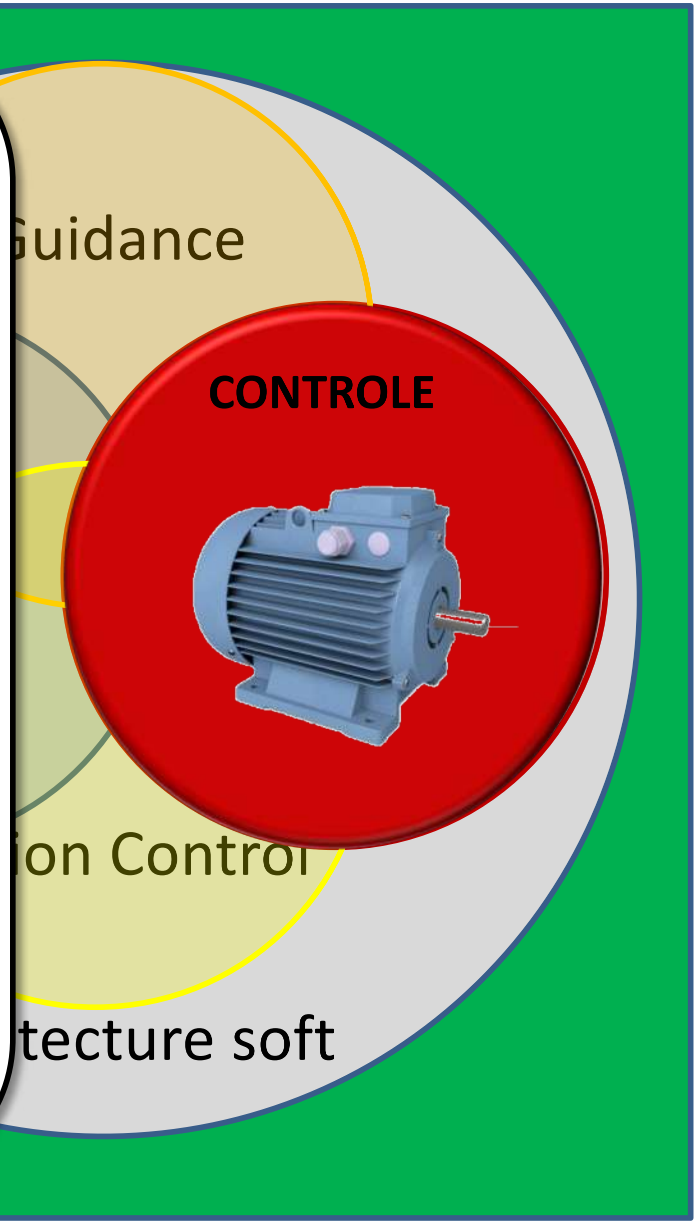


Control

How should I move THAT way ?

The **Control system** generates actuator signals to drive the actual velocity and attitude of the vehicle to the values **provided by the Guidance system**.



Control

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The **Control system** generates actuator signals to drive the actual velocity and attitude of the vehicle to the values **provided by the Guidance system**.

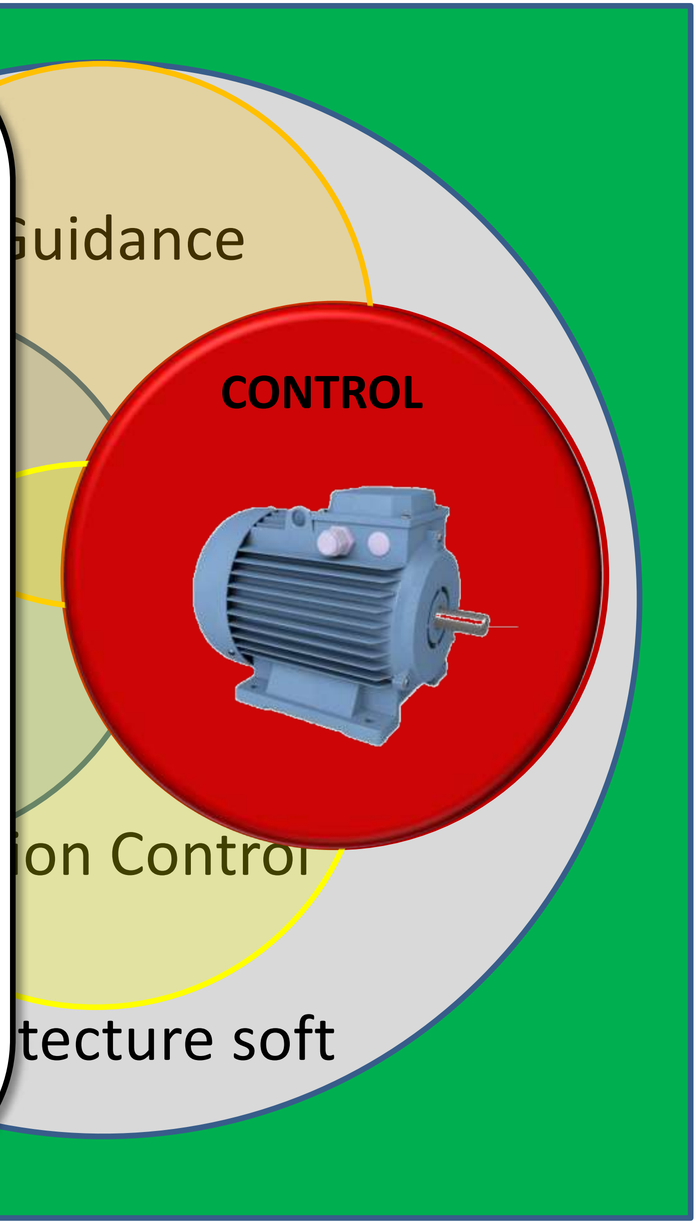
Error function
(Guidance Reference)

↓

CONTROL

↓

Actuation input



Control

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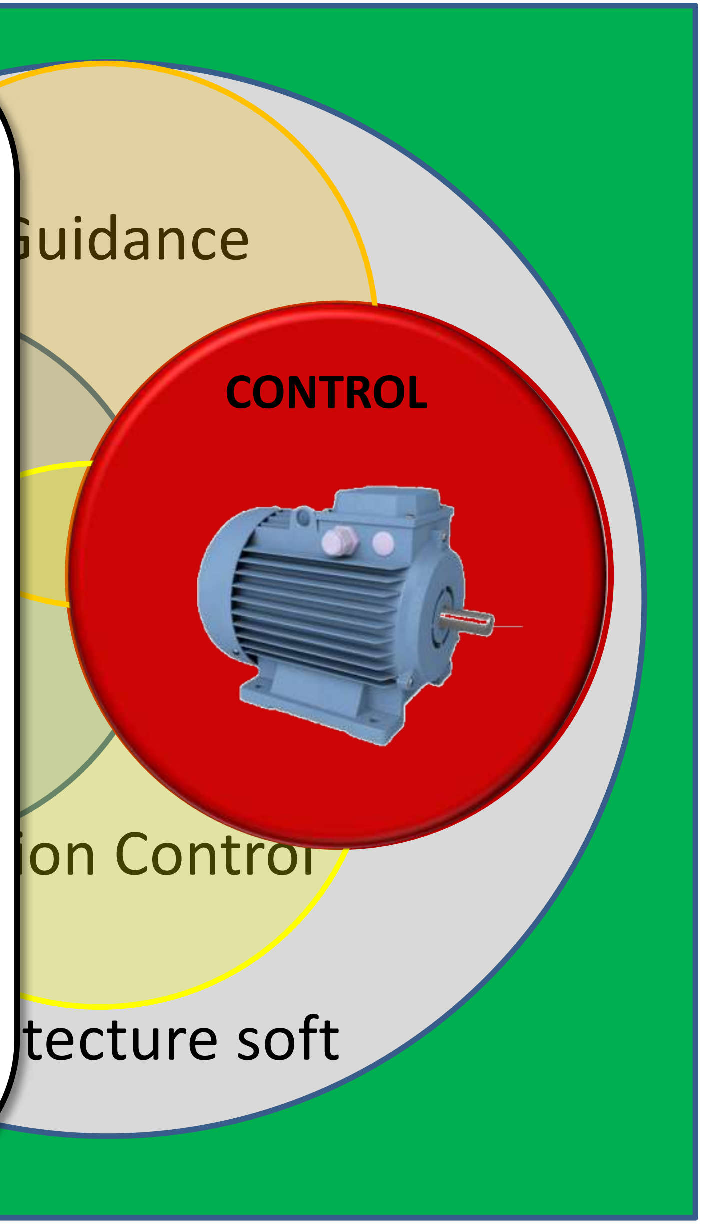
Error function
(Guidance Reference)

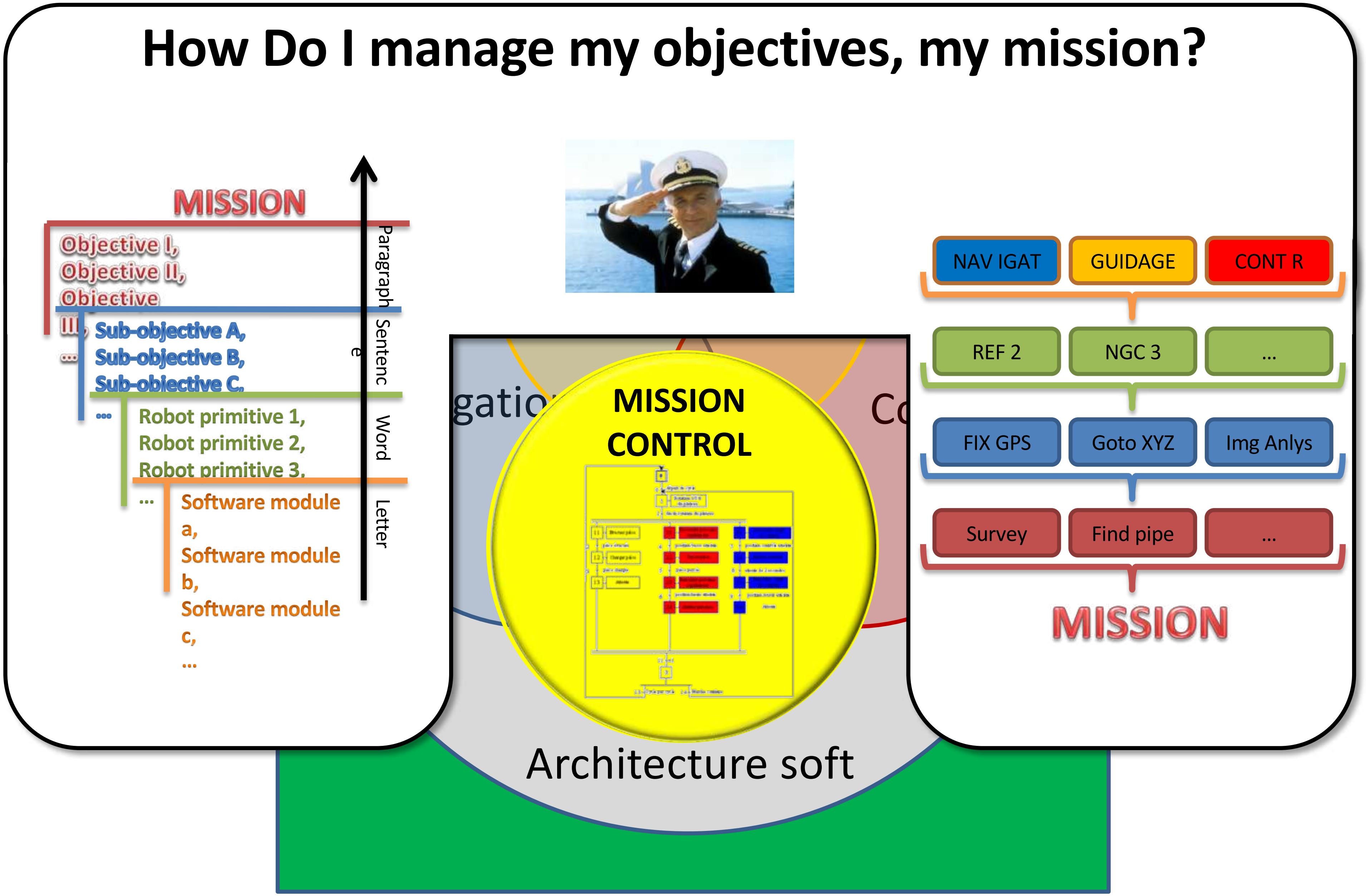
Dynamic Model)

CONTROL & ROBUST SCHEME

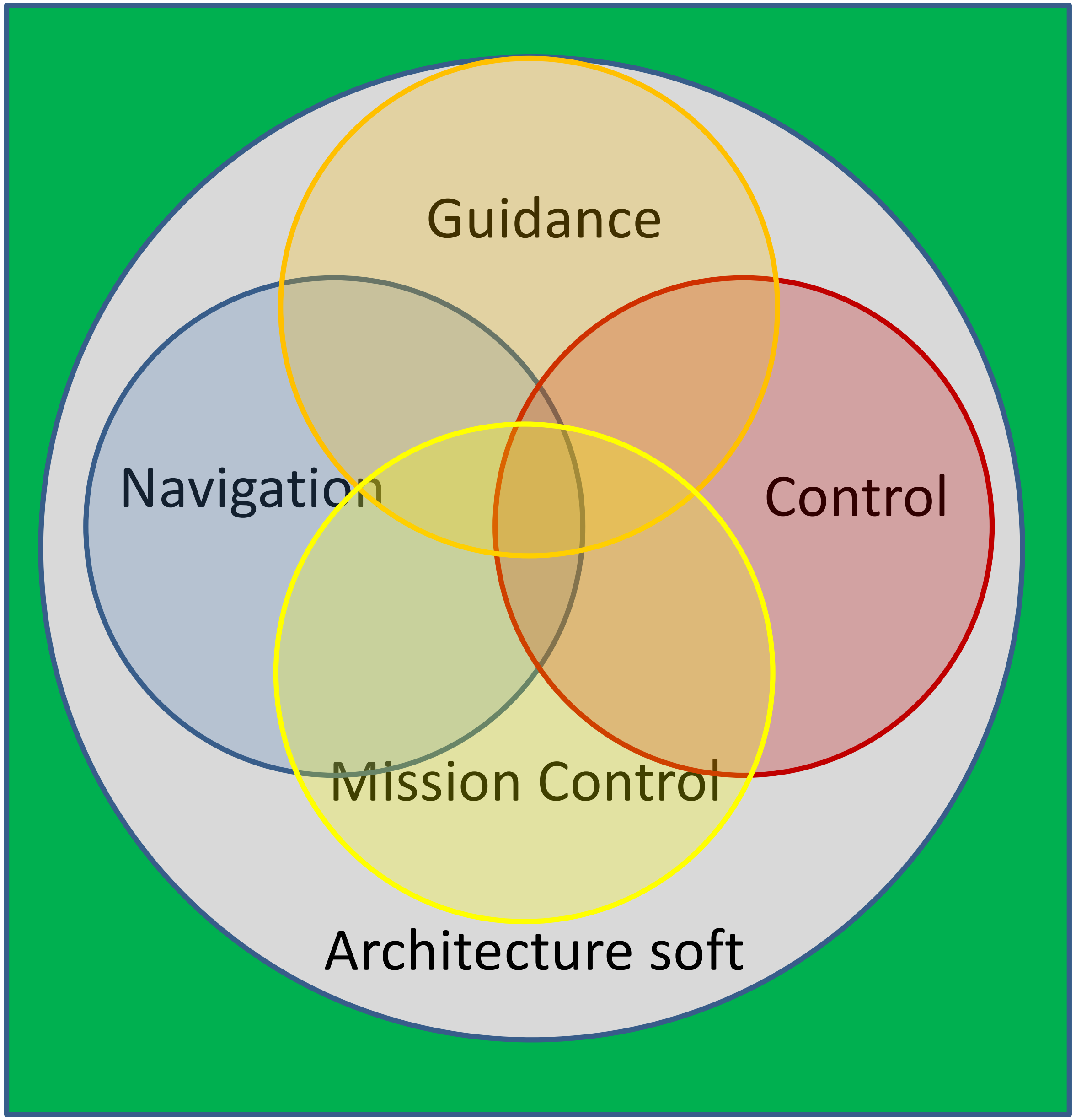
Actuation input



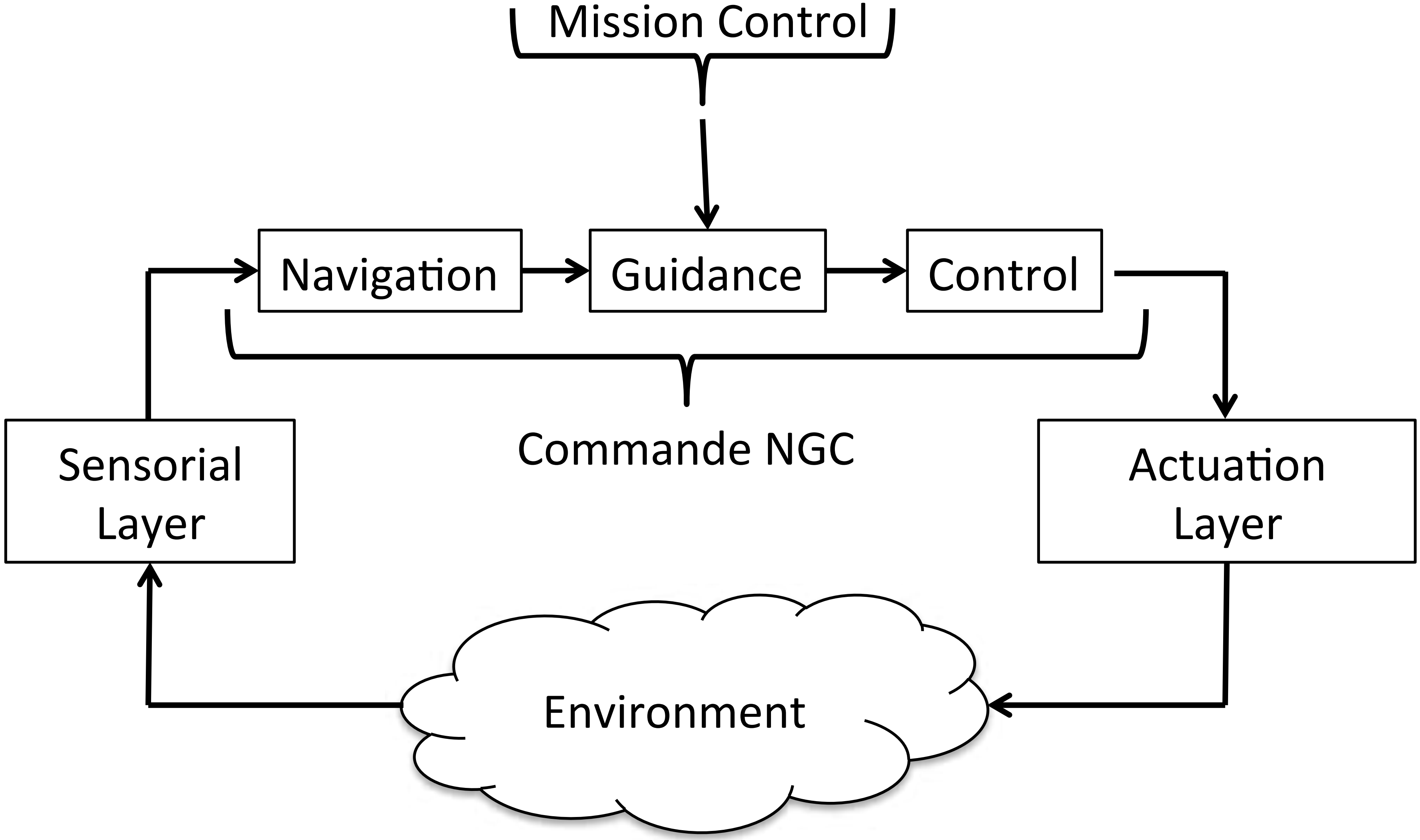
Mission Control



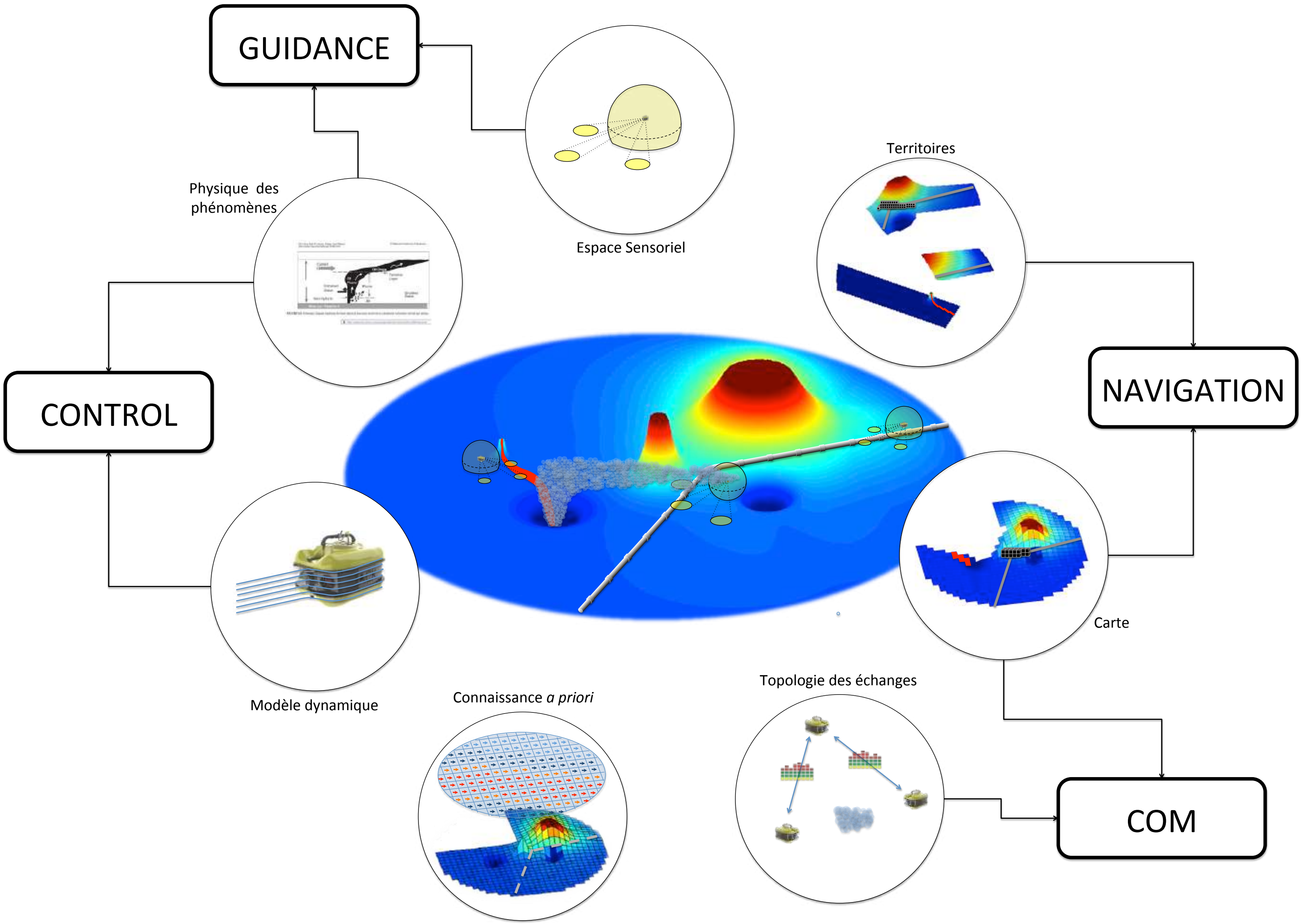
Functions


Guidance
Navigation
Control
Mission Control
Architecture soft

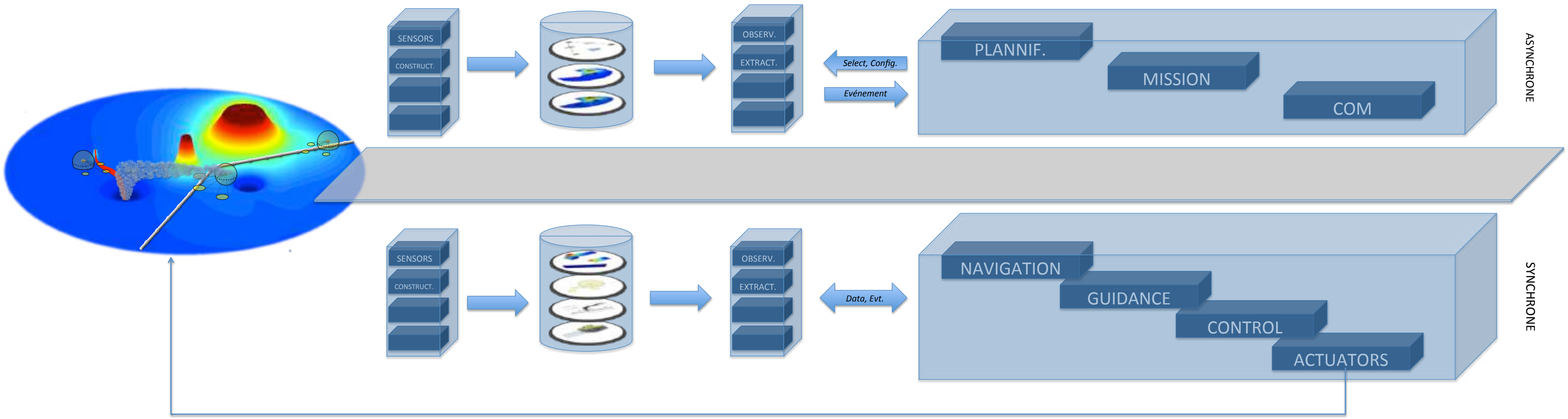
NGC control structure



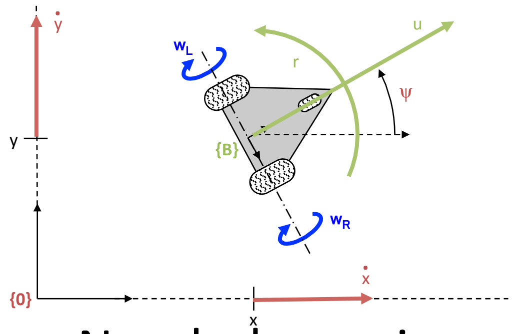
Sensorial Layer



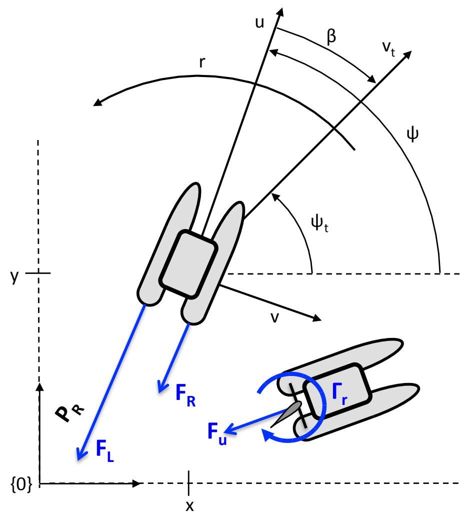
Sensorial Layer

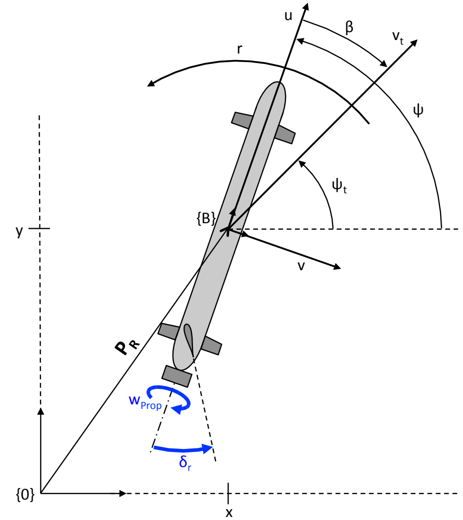


Actuation Layer



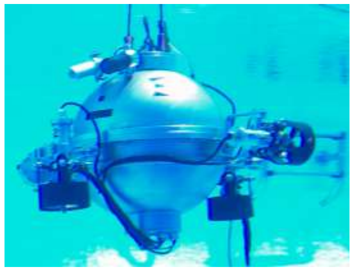
Non-holonomic

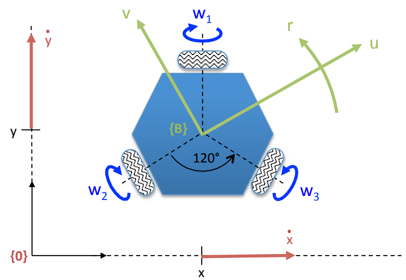


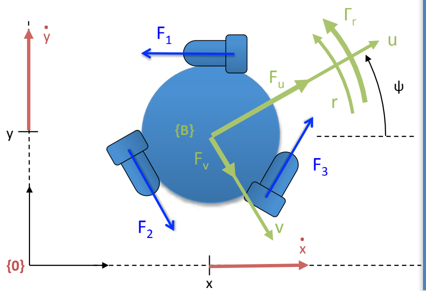


Under-actuated

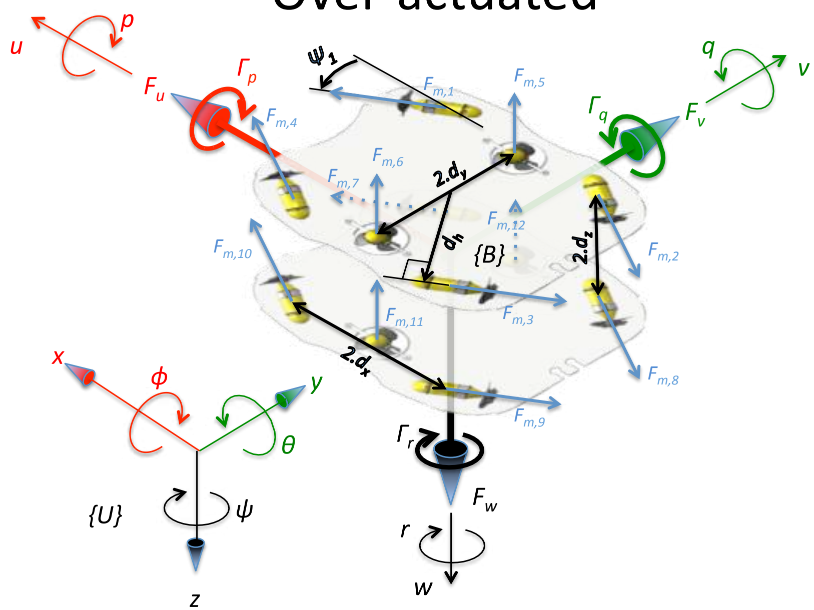
Iso-actuated







Over-actuated



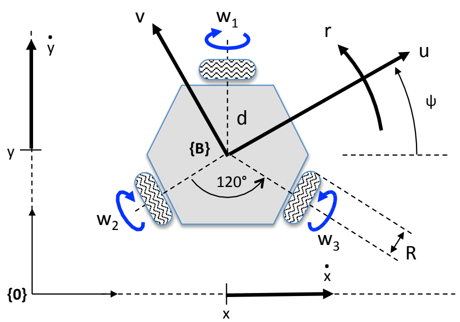
Control functions

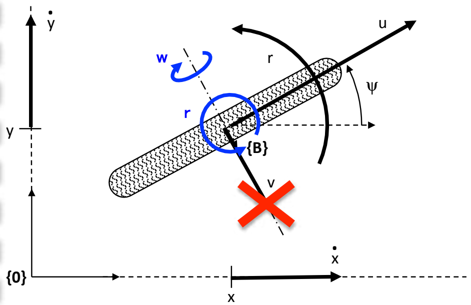
- Mission Control:
  - Define and sequence objectives, sub-objectives…
- Sensorial layer:
  - Build current model of the environment
- Navigation:
  - Estimate system state
- Guidance:
  - Strategy of approach to the objective
- Control:
  - Compute actions to be applied on the environment
- Actuation layer :
  - Compute actuators inputs, manage redundancy (if exists)

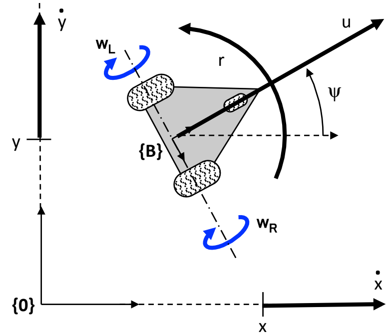
III. Guidance and Control Design

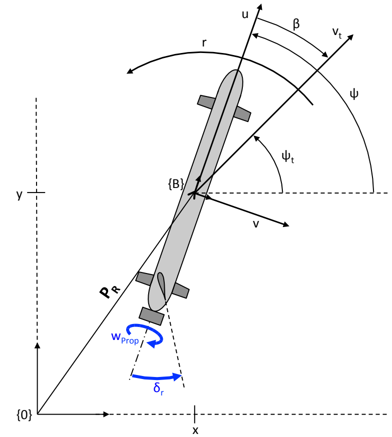
1. Define Kinematic Strategy

The wheel, the Unicycle and the AUV









$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ r \end{bmatrix}$$

Linear approach OK

$$v = \dot{x}\cdot\cos\psi - \dot{y}\cdot\sin\psi = 0$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ r \end{bmatrix}$$

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$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi_t & 0 \\ \sin\psi_t & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_t \\ r+\dot{\beta} \end{bmatrix}$$

Linear approach is inapplicable

$$F_u = m_u\cdot\dot{u} + d_u$$
$$0 = m_v\cdot\dot{v} + m_{ur}\cdot u\cdot r + d_v$$
$$\Gamma = m_r\cdot\dot{r} + d_r$$

$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}}_{\chi} = \underbrace{\frac{-1}{R}\cdot\begin{bmatrix} -\sin\alpha_1 & \cos\alpha_1 & d \\ -\sin\alpha_2 & \cos\alpha_2 & d \\ -\sin\alpha_3 & \cos\alpha_3 & d \end{bmatrix}}_{\mathbf{M}_A^{-1}} \cdot \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{\upsilon}$$

$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} w \\ r \end{bmatrix}$$

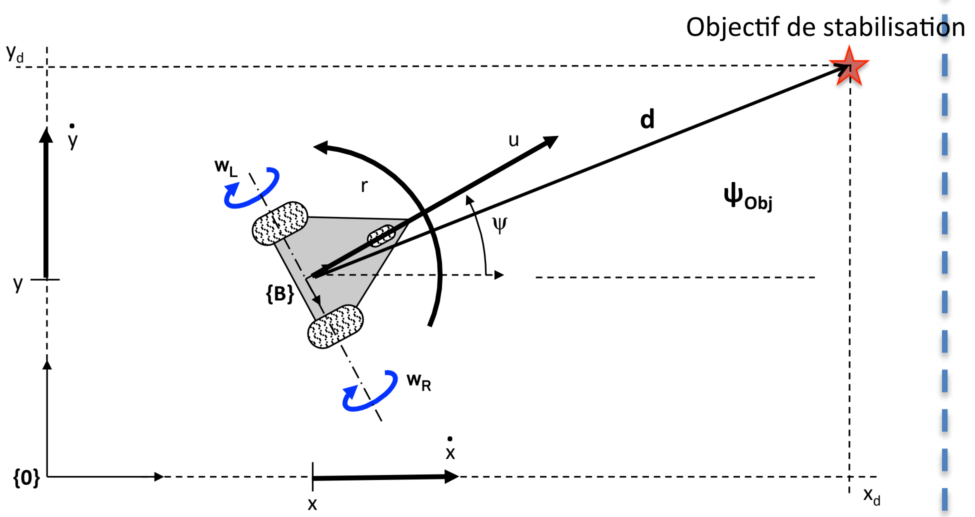
$$\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} \frac{R}{2} & \frac{R}{2} \\ \frac{R}{2\cdot L} & \frac{-R}{2\cdot L} \end{bmatrix} \cdot \begin{bmatrix} w_l \\ w_r \end{bmatrix}$$

$$\mathbf{F}_B = \begin{bmatrix} F_u \\ \Gamma \end{bmatrix} = \mathbf{M}_A\cdot\mathbf{F}_m(w_{prop},\delta)$$

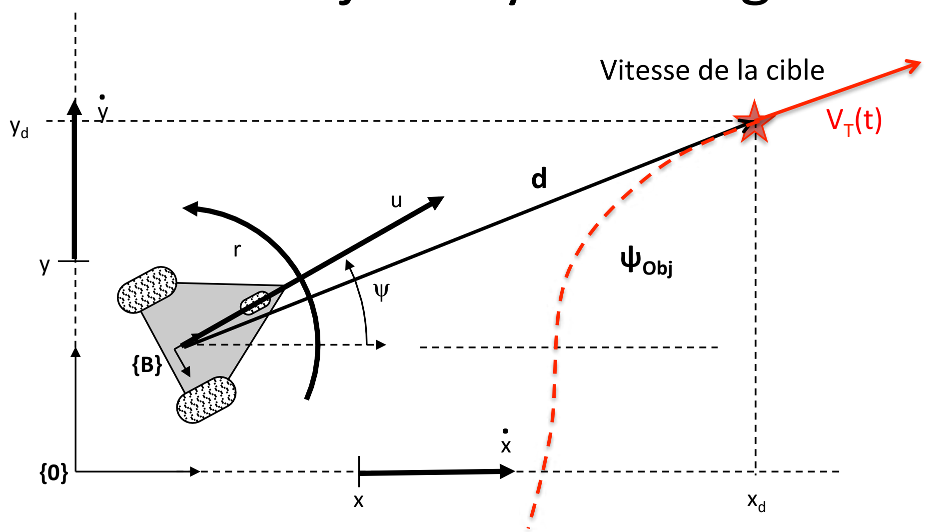
The transposition of Unicycle solution to AUV depends on the system shape. $\frac{m_{ur}}{m_v} < 1$

2. Choose movement control type

Stabilisation



Trajectory Tracking

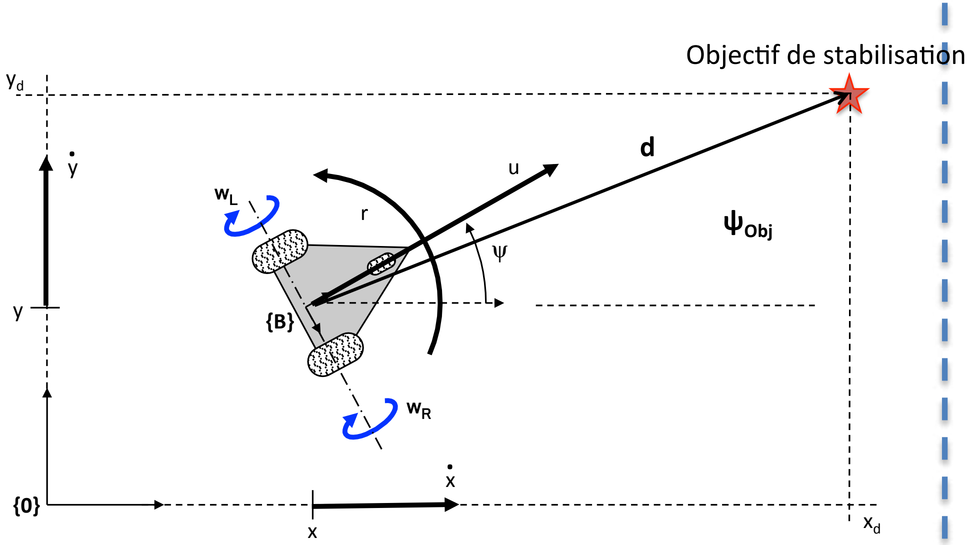


There is NO continuous control law that could stabilise a nonholonomic system [BROCKETT 83]

- Singular objective
- A small error induces a large manoeuvre
- The linearized model is not controllable
- Trajectory : Space & **<u>Time</u>** reference

2. Choose movement control type

Stabilisation



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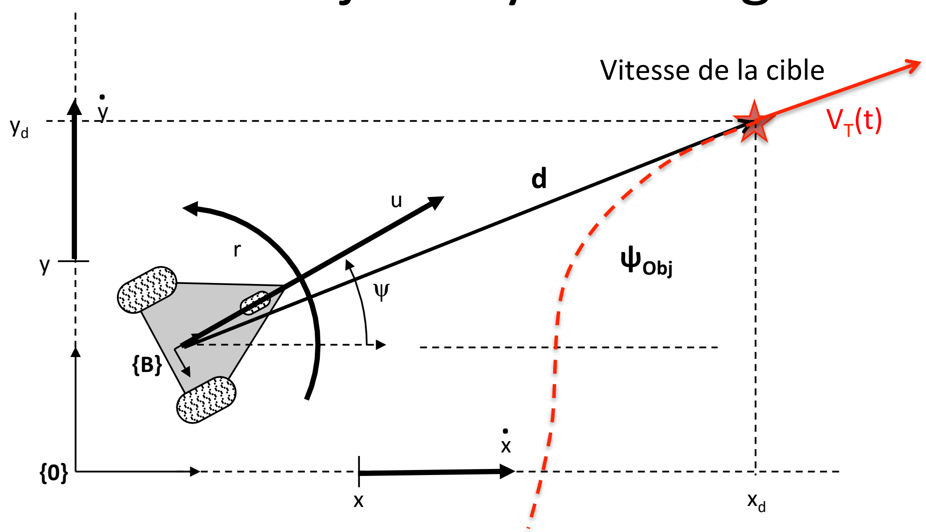

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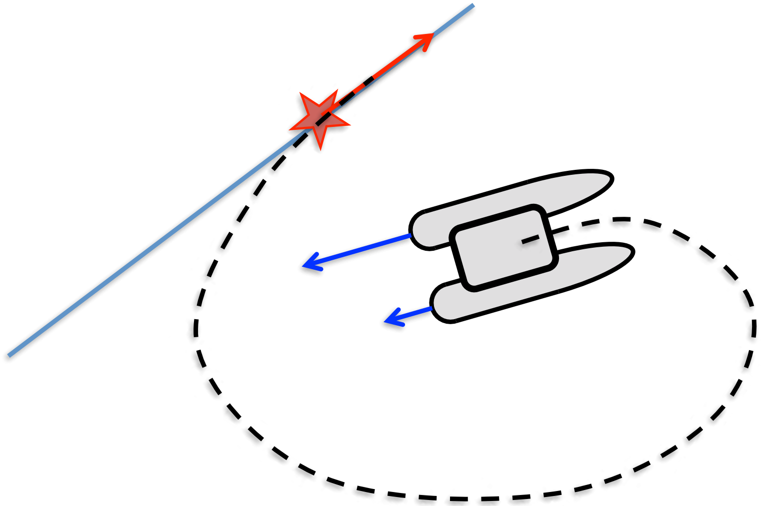

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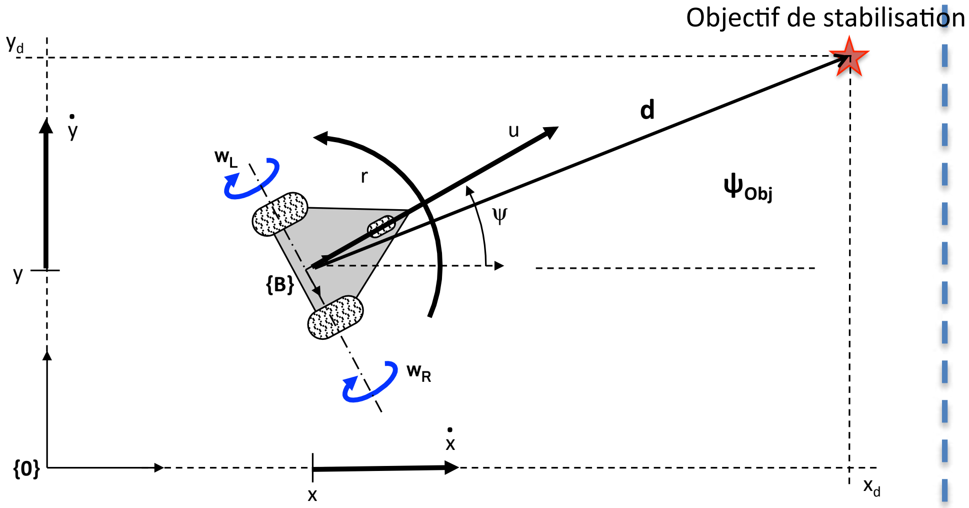

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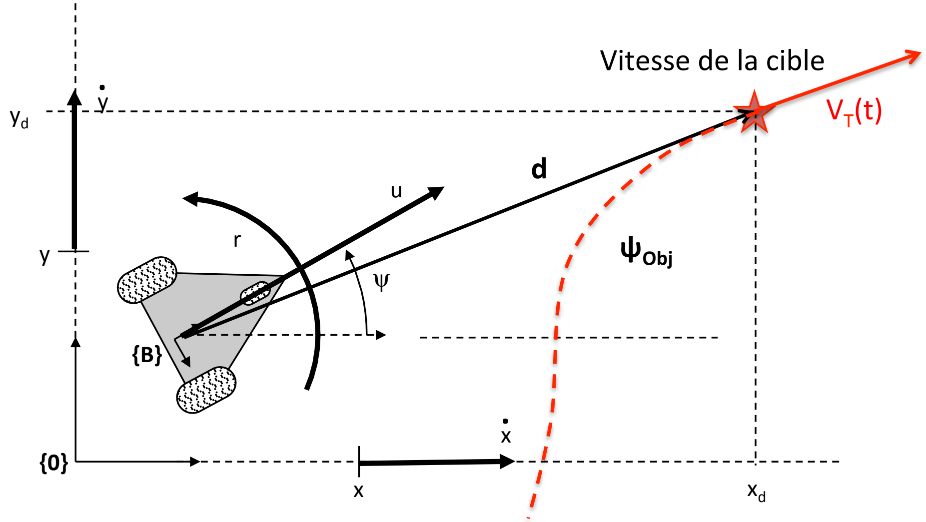

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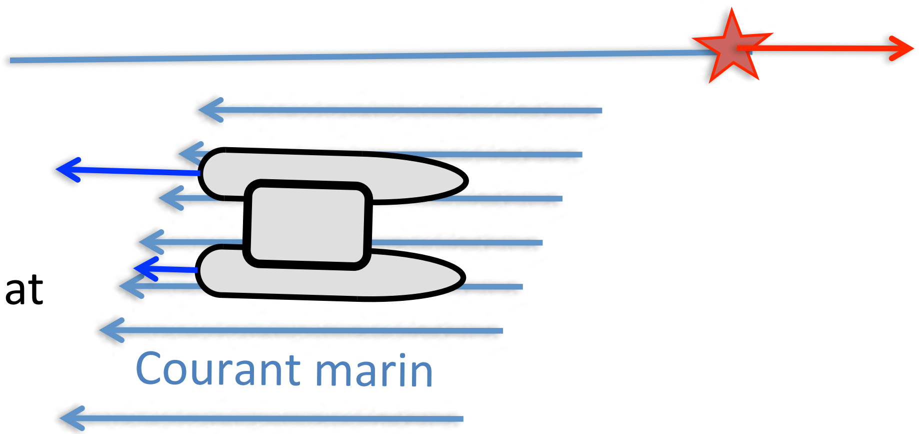
- Actuators may easily be pushed to saturation



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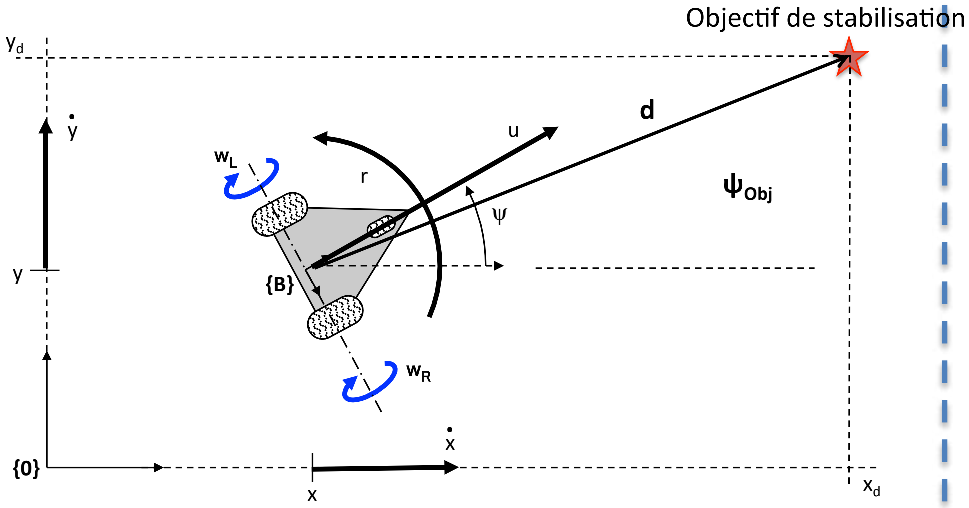
Stabilisation



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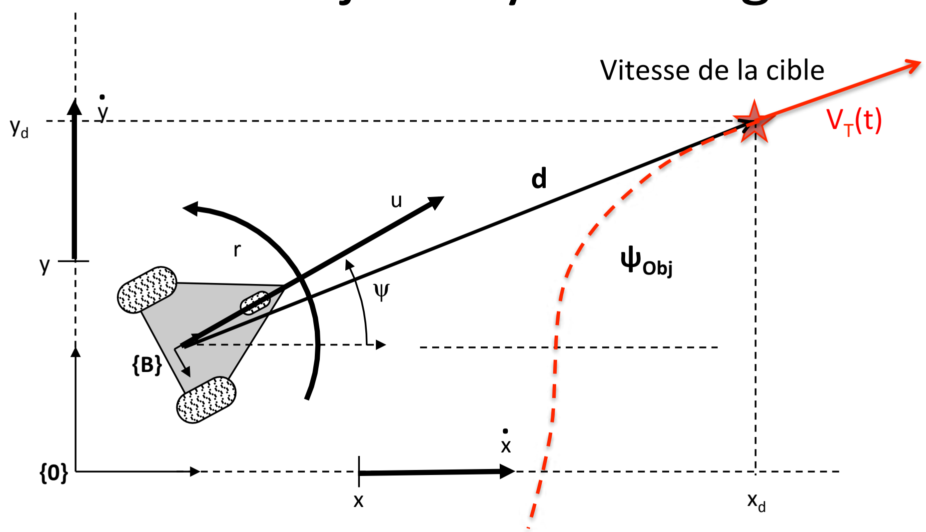

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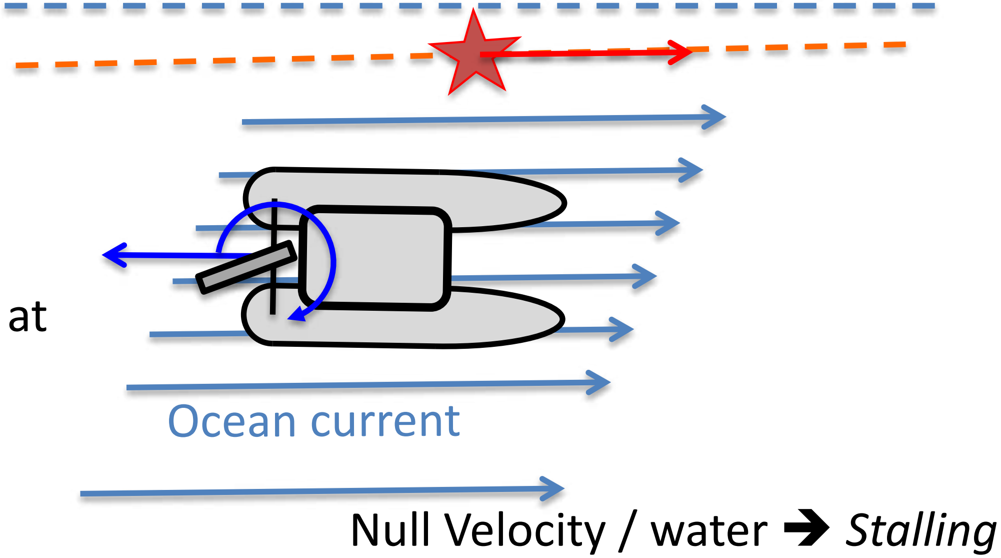
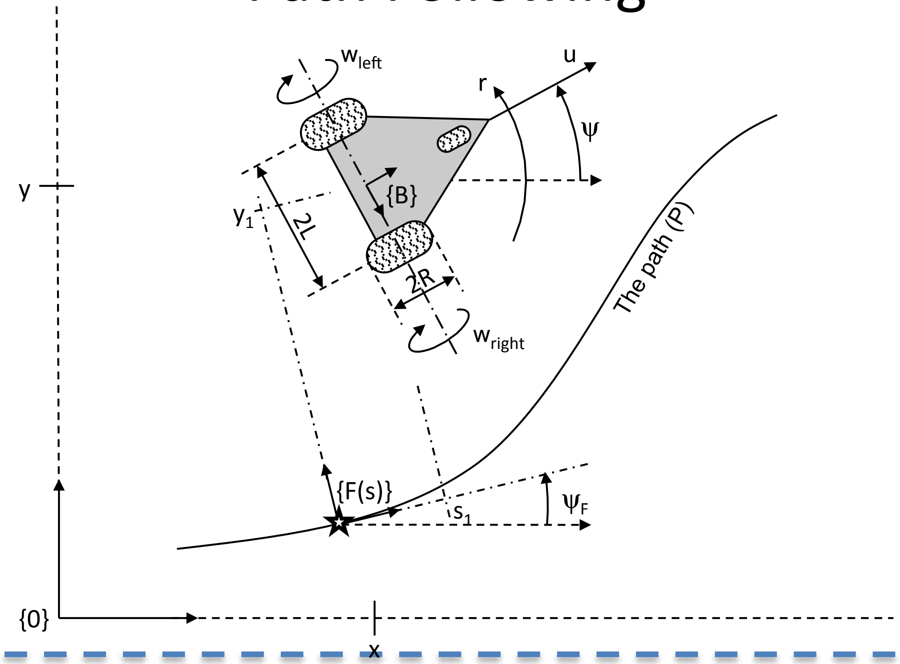
- Danger of **Stalling**



2. Choose movement control type

Path Following



- Decoupled control u and r
- Remove temporal constraint
- Autonomous system
- ‘smooth’ convergence to the path
- Keep control on actuator saturation
- Global and Uniform Asymptotic Convergence
- Can be extended to RDV tracking
- Can be extended to formation keeping

~~There is NO continuous control law that could stabilise a nonholonomic system [BROCKETT 83]~~

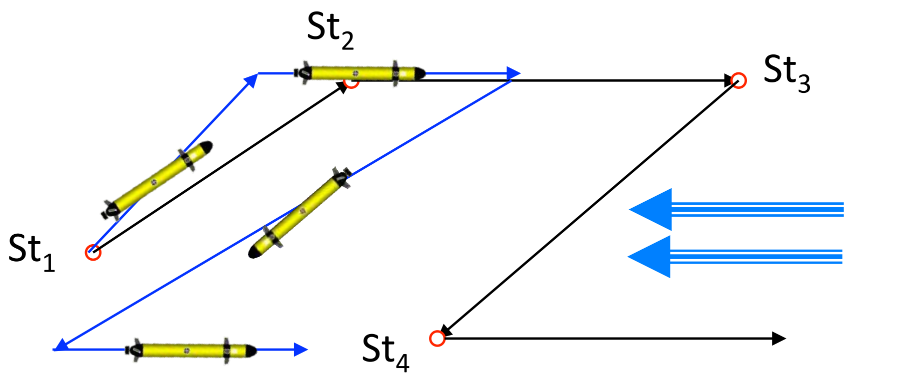
- ~~Singular objective~~
- ~~A small error induces a large manoeuvre~~
- The linearized model is not controllable
- Path Following : Space & ~~**Time**~~ reference
- ~~The vehicle may turn back in its attemp to be at a desired reference at a prescribed time~~
- ~~Actuators may easily be pushed to saturation~~
- ~~Danger of **Stalling**~~

3. Choose Guidance Strategy

St_i	u_i	$\Psi_{d,i}$	t_i
St_1	1m/s	40°	30s
St_2	1m/s	90°	30s
St_3	1m/s	220°	30s
St_4	1m/s	40°	30s

Set-points

(heading – Duration)

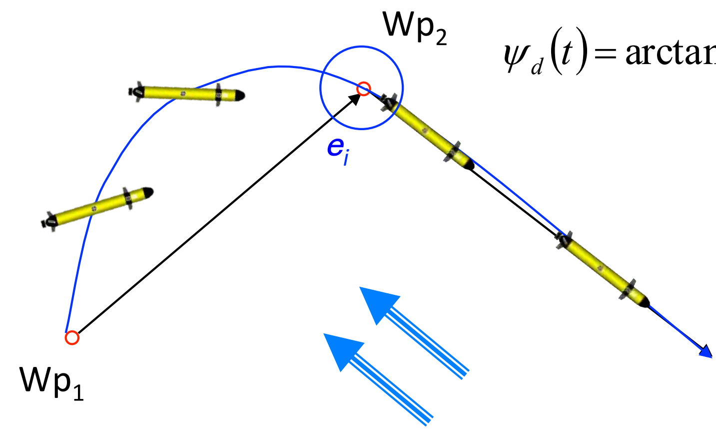


Waypoints

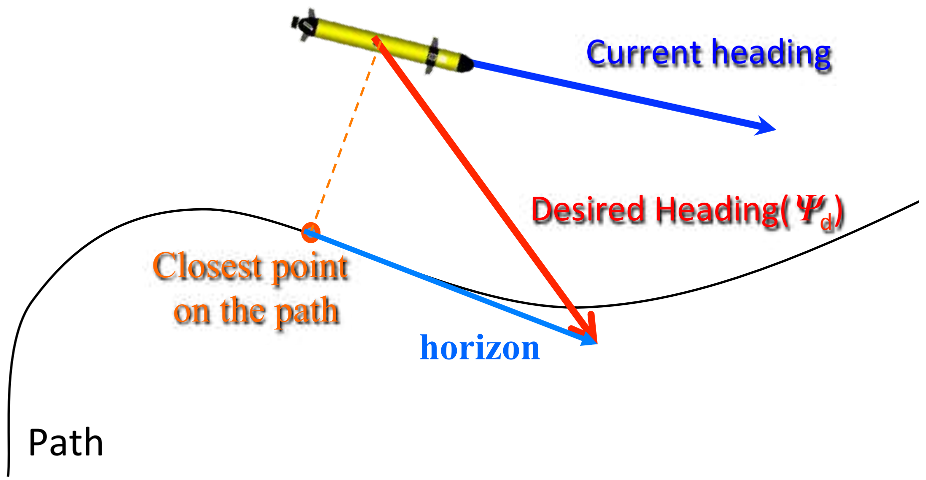
(geo-referenced)

Wp_i	u_i	x_i	y_i	e_i
Wp_1	1m/s	10m	20m	.1m
Wp_2	1m/s	0m	40m	.2m

$$\psi_d(t) = \arctan\left(\frac{x_k - x(t)}{y_k - y(t)}\right)$$

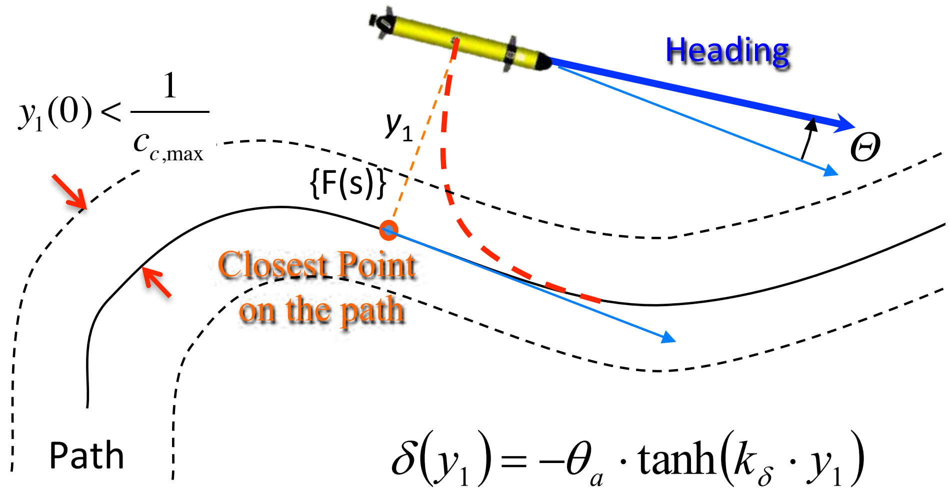


Line of sight



Path Following

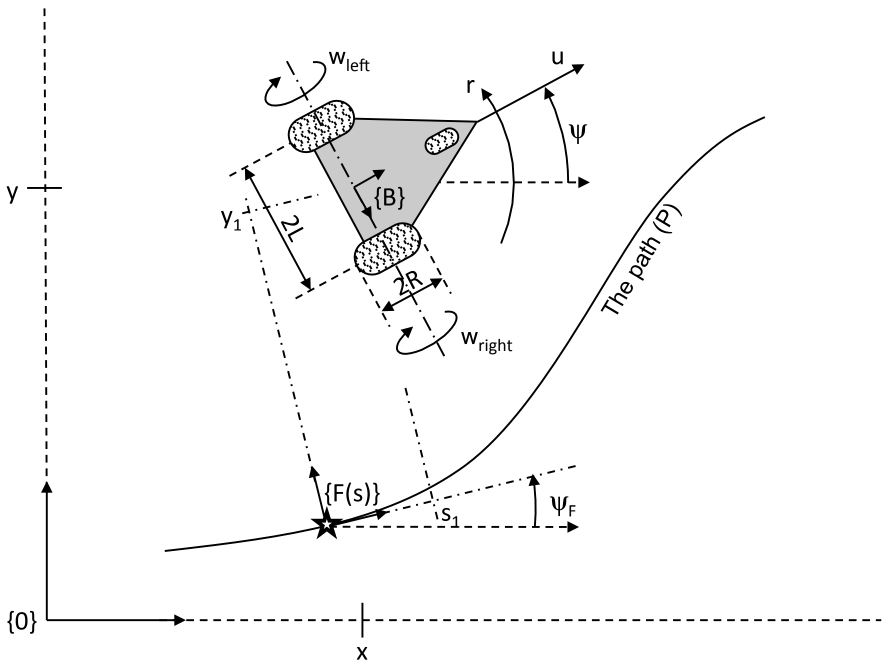
$$y_1(0) < \frac{1}{c_{c,\max}}$$



$$\delta(y_1) = -\theta_a \cdot \tanh(k_\delta \cdot y_1)$$

3. Choose Guidance Strategy

Virtual Target Path Following



3.a Express kinematic model in the Frenet frame

$$\dot{s}_1 = -\dot{s}\cdot\left(1 - c_c \cdot y_1\right) + u\cdot\cos\theta$$

$$\dot{y}_1 = -c_c\cdot\dot{s}\cdot s_1 + u\cdot\sin\theta$$

$$\dot{\theta} = r - c_c\cdot\dot{s}$$

3.b Choose an approach angle

$$\delta(y_1) = -\text{sign}(u)\cdot\arctan(K_\delta\cdot y_1)$$

3.c Consider the Lyapunov Candidate

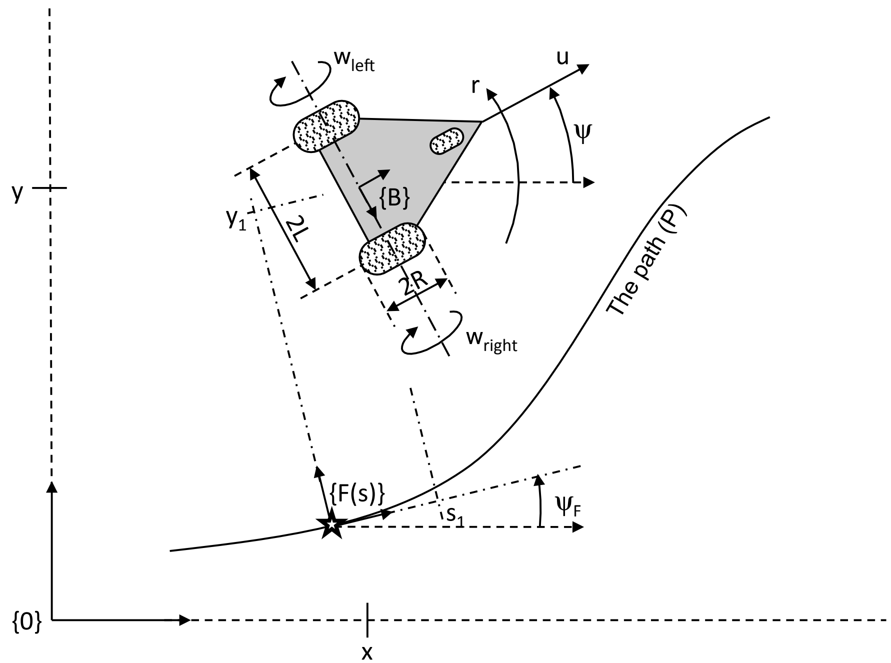
$$V = \frac{1}{2}\cdot\left(s_1^2 + y_1^2\right) + \frac{1}{2\cdot\gamma}\cdot(\theta-\delta)^2 > 0$$

3.c.1 Derivate V

$$\begin{aligned}\dot{V}_1 &= s_1\dot{s}_1 + y_1\dot{y}_1 + \frac{1}{\gamma}(\theta-\delta)(\dot{\theta}-\dot{\delta}) \\ &= s_1(v\cos\theta - \dot{s}(1-c_c y_1) - \dot{s}c_c y_1) + y_1 v\sin\theta + \frac{1}{\gamma}(\theta-\delta)(\dot{\theta}-\dot{\delta}) \\ &= s_1(v\cos\theta - \dot{s}) + y_1 v\sin\delta + \frac{1}{\gamma}(\theta-\delta)\left(\dot{\theta}-\dot{\delta}+\gamma y_1 v\frac{\sin\theta-\sin\delta}{\theta-\delta}\right)\end{aligned}$$

3. Choose Guidance Strategy

Virtual Target Path Following



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3.c.2 Choose

$$\begin{cases} \dot{s} = u\cdot\cos\theta + k_1\cdot s_1 \\ r = \dot{\delta} - \gamma\cdot y_1\cdot u\cdot\dfrac{\sin\theta - \sin\delta}{\theta-\delta} - k_2\cdot(\theta-\delta) + c_c\cdot\dot{s} \\ u = u_d > 0 \end{cases}$$

$$\dot{V}_1 = s_1\dot{s}_1 + y_1\dot{y}_1 + \frac{1}{\gamma}(\theta-\delta)(\dot{\theta}-\dot{\delta})$$

$$= s_1(v\cos\theta - \dot{s}(1-c_c y_1) - \dot{s}c_c y_1) + y_1 v\sin\theta + \frac{1}{\gamma}(\theta-\delta)(\dot{\theta}-\dot{\delta})$$

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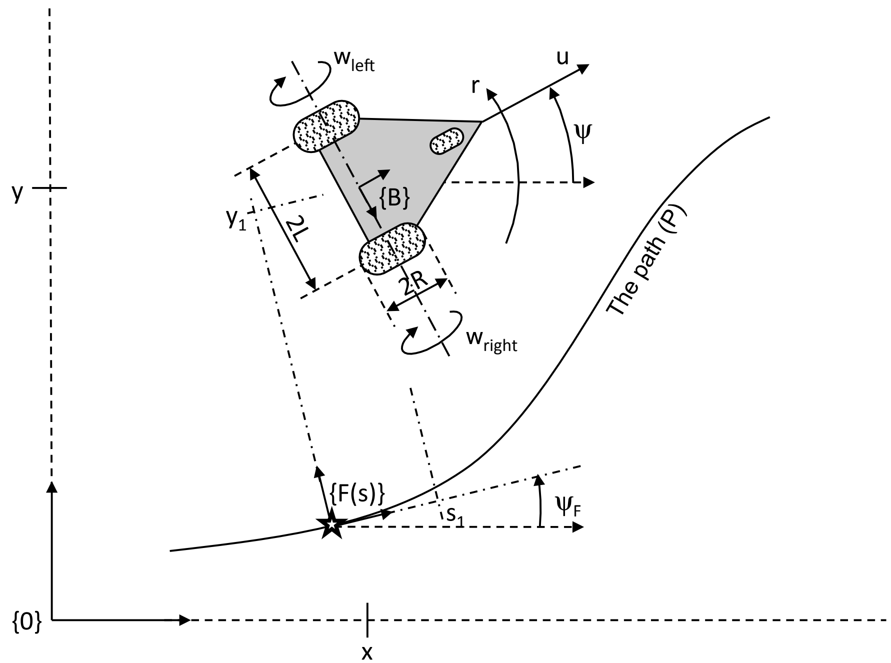
3. Choose Guidance Strategy

Virtual Target Path Following



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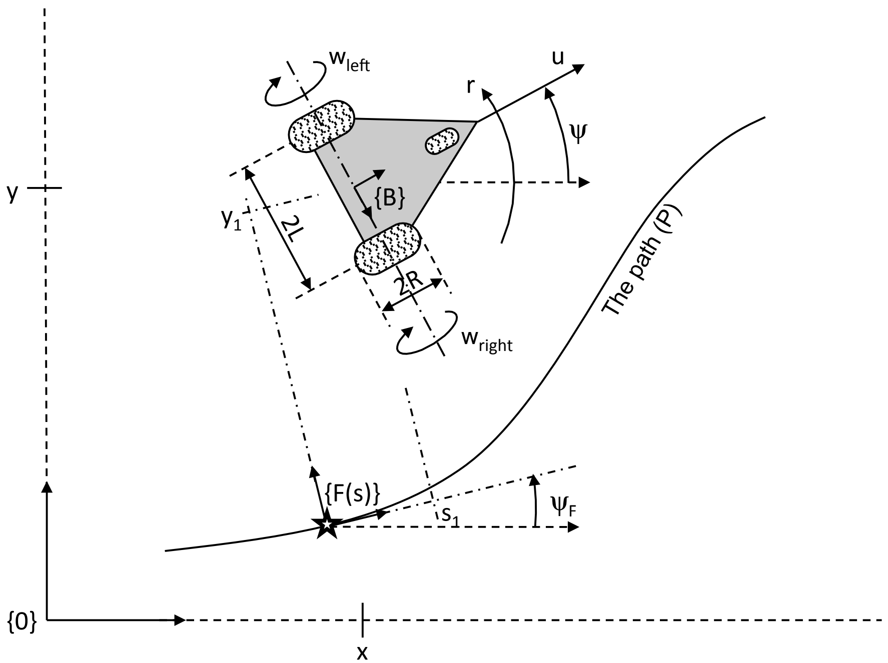
3.d Then, the following control:

$$\begin{cases} \dot{s} = u\cdot\cos\theta + k_1\cdot s_1 \\ r = \dot{\delta} - \gamma\cdot y_1\cdot u\cdot\dfrac{\sin\theta - \sin\delta}{\theta-\delta} - k_2\cdot(\theta-\delta) + c_c\cdot\dot{s} \\ u = u_d > 0 \end{cases}$$

$$\Rightarrow \left\{ \begin{array}{c} \dot{V} \le 0 \\ V(\infty) = \infty \end{array} \right\} \text{ and } \ddot{V} \text{ bounded} \xrightarrow[\text{Barbalat's Lemma}]{} \lim_{t\to\infty}(y_1, s_1, \theta) = 0$$

3. Choose Guidance Strategy

Virtual Target Path Following



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3.b Choose an approach angle

$$\delta(y_1) = -\mathrm{sign}(u) \cdot \arctan(K_\delta \cdot y_1)$$

3.c Consider the Lyapunov Candidate

$$V_1 = \frac{1}{2}\cdot\left(s_1^2 + y_1^2\right), \text{ then } V_2 = (\theta - \delta)^2\Big|_{V_1 = 0} > 0$$

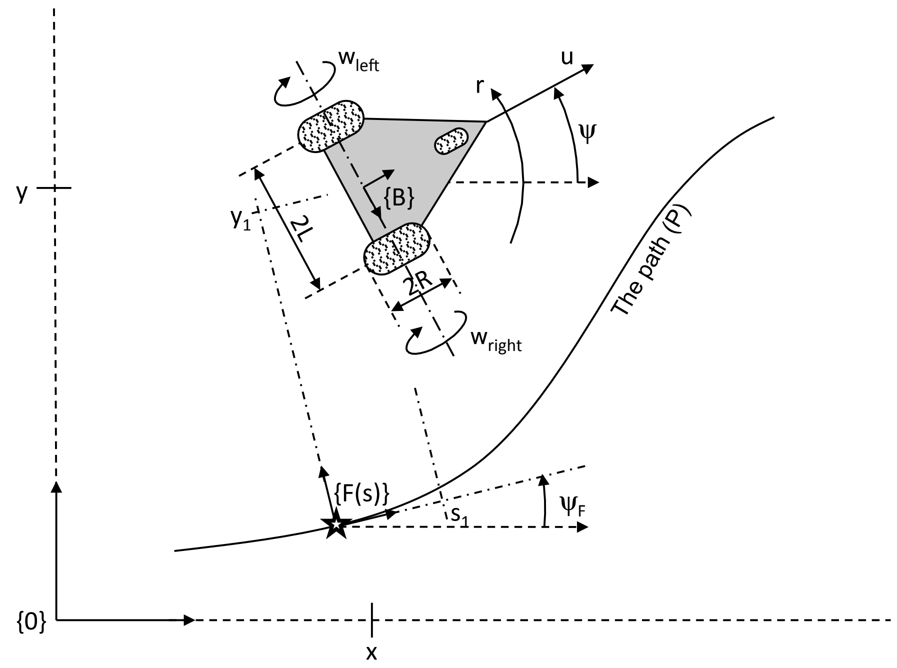
3.d Then, the following control:

$$\begin{cases} \dot{s} = u \cdot \cos\theta + k_1 \cdot s_1 \\ r = \dot{\delta} - \gamma \cdot y_1 \cdot u \cdot \cancel{\dfrac{\sin\theta - \sin\delta}{\theta - \delta}} - k_2 \cdot (\theta - \delta) + c_c \cdot \dot{s} \\ u = u_d > 0 \end{cases}$$

$$\Rightarrow \left\{ \begin{array}{c} \dot{V}_1 \le 0 \\ \dot{V}_2\Big|_{V_1=0} < 0 \end{array} \right\} \text{ and (if S is autonomous)} \xrightarrow[\text{LaSalle Inv. Princ.}]{\text{Barbalat's Lemma \&}} \lim_{t\to\infty}(y_1, s_1, \theta) = 0$$

3. Choose Guidance Strategy

Virtual Target Path Following



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3.d Then, the following control:

$$\begin{cases} \dot{s} = u\cdot\cos\theta + k_1\cdot s_1 \\ r = \dot{\delta} - k_2\cdot(\theta-\delta) + c_c\cdot\dot{s} \\ u = u_d > 0 \end{cases}$$

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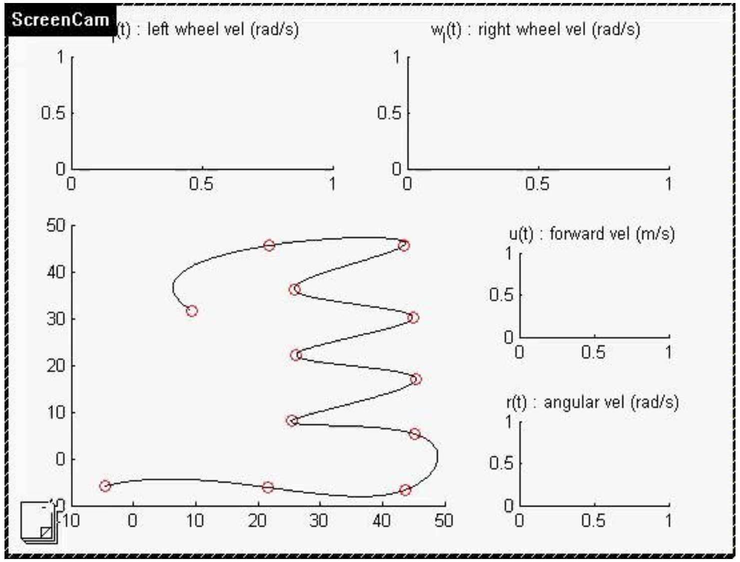
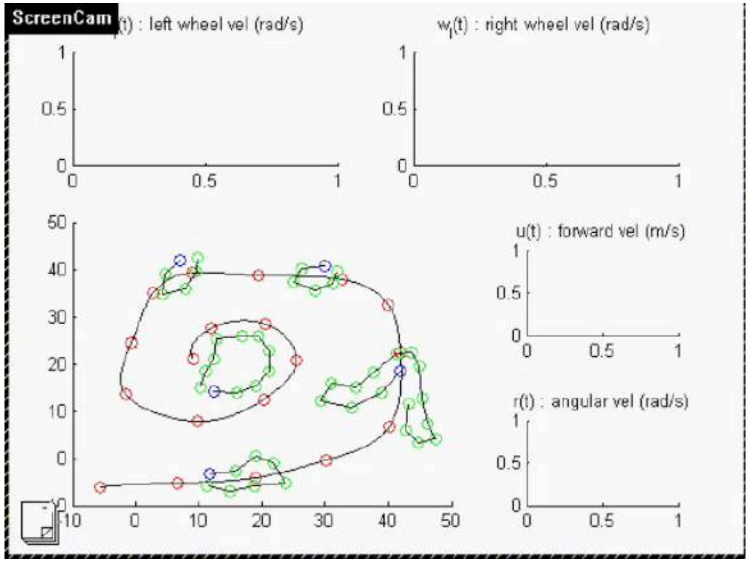
Extentions

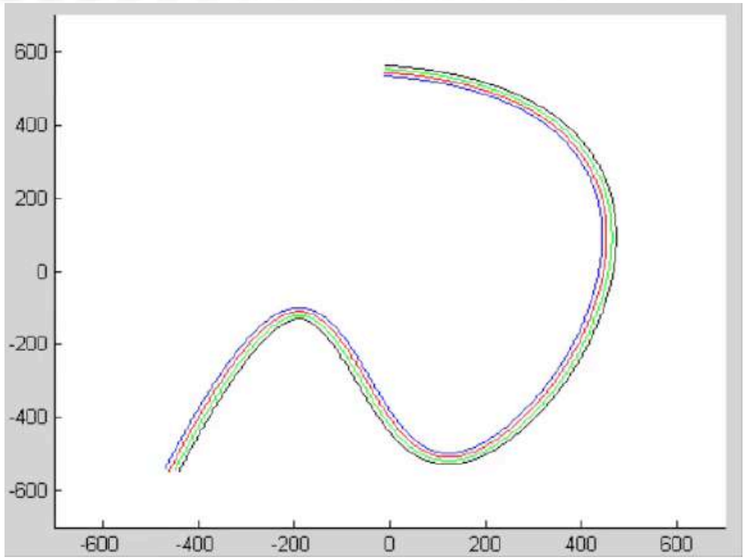
Virtual Target Path Following



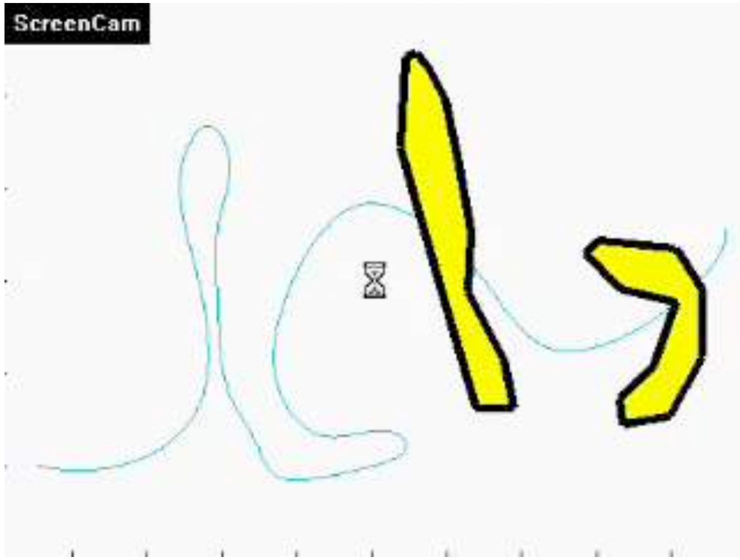
PF with AS



PF with AS and OA

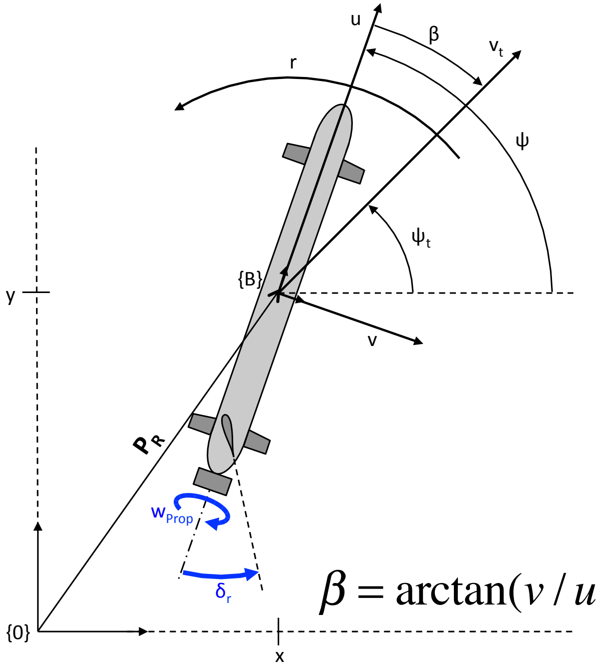


Multi-PF with CA



Multi-PF with OA and MST

4. Dynamics Backstepping



$$\beta = \arctan(v/u)$$

4.a Augment system model with its dynamic state

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi_t & 0 \\ \sin\psi_t & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_t \\ r+\dot{\beta} \end{bmatrix}$$

$$F_u = m_u \cdot \dot{u} + d_u$$
$$0 = m_v \cdot \dot{v} + m_{ur} \cdot u \cdot r + d_v$$
$$\Gamma = m_r \cdot \dot{r} + d_r$$

4.b Consider previous kinematic solution as a reference

$$U_{KIN} \begin{cases} \dot{s} = u \cdot \cos\theta + k_1 \cdot s_1 \\ r_{KIN} = \dot{\delta} - \dot{\beta} - k_2 \cdot (\theta - \delta) + c_c(s) \cdot \dot{s} \\ u_{KIN} = u_d > 0 \end{cases}$$

4.c Consider Lyapunov Candidate

$$V = \frac{1}{2} \cdot \left(s_1^2 + y_1^2\right) + \frac{1}{2 \cdot \gamma} \cdot (\theta - \delta)^2 + \frac{1}{2} \cdot \left(r - r_{KIN}\right)^2 + \frac{1}{2} \cdot \left(u - u_{KIN}\right)^2 > 0$$

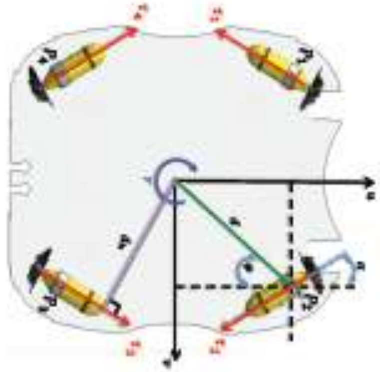
4.d Then the following control

$$U_{DYN} \begin{cases} \dot{s} = u \cdot \cos\theta + k_1 \cdot s_1 \\ \Gamma = m_r \cdot \dfrac{\left(f_{\dot{r}_{DYN}} - k_3 \cdot \left(r - r_{KIN}\right) - k_4 \cdot (\theta - \delta)\right)}{1 - \dfrac{m_{ur}}{m_v} \cdot \cos^2\beta} + d_r \\ F_u = m_u \cdot \left(\dot{u}_d - k_5 \cdot \left(u - u_d\right)\right) + d_u \end{cases} \Rightarrow \begin{cases} \dot{V} \le 0 \\ \ddot{V} \text{ bounded} \end{cases} \xrightarrow[\text{LaSalle Inv Princ.}]{\text{Barbalat's Lemma}} \lim_{t \to \infty} \left(y_1, s_1, \theta, u - u_d\right) = 0$$

$$f_{\dot{r}_{DYN}} = \ddot{\delta} - k_3 \cdot \left(\dot{\theta} - \dot{\delta}\right) - k_4 \cdot (\theta - \delta) + c_c(s) \cdot \ddot{s} + \frac{\partial c_c}{\partial s} \cdot \dot{s}^2$$

4. Develop Adaptive / Switching scheme

6. Actuation layer robustness



6.a Consider the redundant system

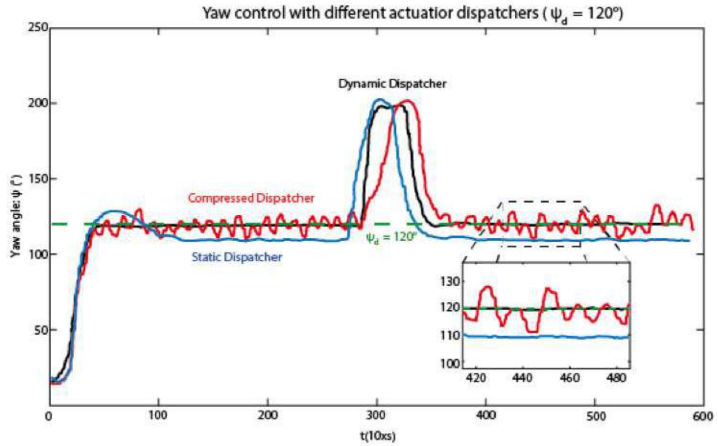
$$\mathbf{F_B} = \mathbf{A} \cdot \mathbf{F_m}$$

(Concentrator)

$$\mathbf{A} = \begin{bmatrix} -.866 & -.866 & .866 & .866 \\ -.5 & .5 & .5 & -.5 \\ .226 & -.226 & .226 & -.226 \end{bmatrix}$$

6.b Consider the yaw control (simple PD)

$$\mathbf{F}_{\mathbf{B}}^{d} = \begin{bmatrix} F_u \\ F_v \\ \Gamma_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k_1 \cdot (\psi_d - \psi) - k_2 \cdot r \end{bmatrix}$$

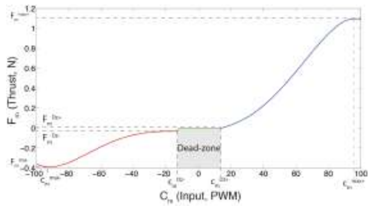


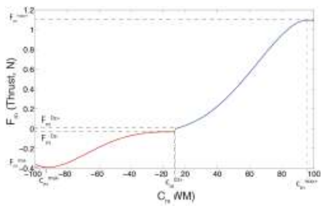
6.c Compute Actuator Forces (classic Moore-Pensose Pseudo-inverse)

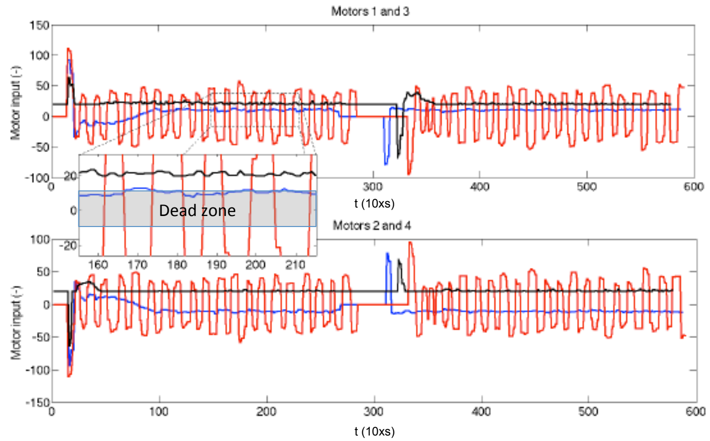
$$\mathbf{F_m} = \mathbf{A}^{+} \cdot \mathbf{F_B^d}$$

(Classic Dispatcher)

6.d Classic reflex : Compress dead-zone







6. Actuation layer robustness

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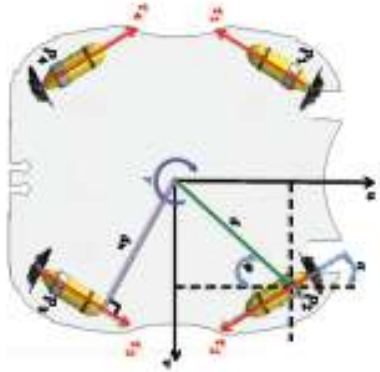

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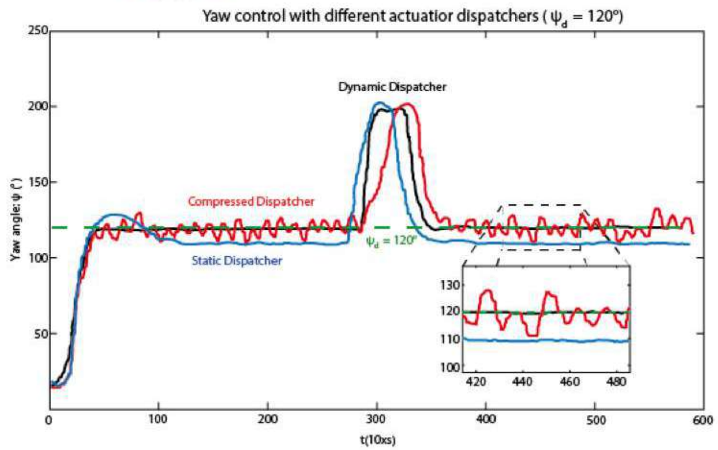


6.c Use redundancy

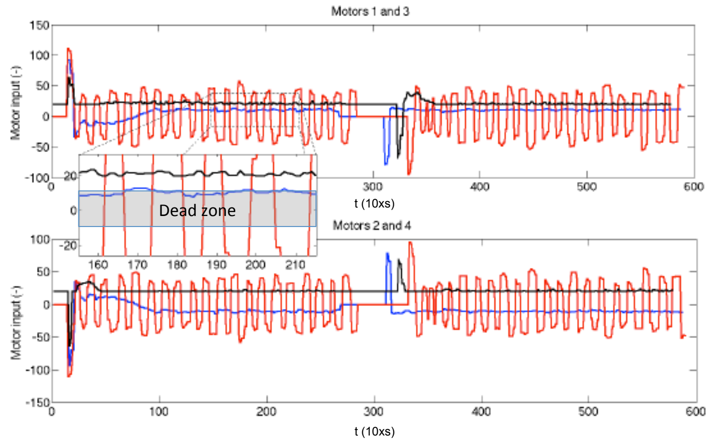
$$\mathbf{F_m} = \underbrace{\begin{bmatrix} \mathbf{A}^+ & \mathbf{M_m} \end{bmatrix}}_{\text{(Dispatcher)}} \cdot \begin{bmatrix} \mathbf{F_B^d} \\ r_m \end{bmatrix}$$

$$\mathbf{M_m} \in \ker(\mathbf{A}) \Rightarrow \mathbf{A} \cdot \mathbf{M_m} = \mathbf{0}$$

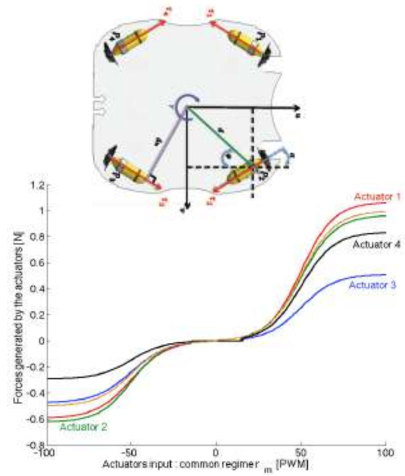
$$\mathbf{F}_B = \mathbf{A} \cdot \mathbf{A}^+ \cdot \mathbf{F_B^d} + \mathbf{A} \cdot \mathbf{M_m} \cdot r_m$$

$$\mathbf{F}_B = \mathbf{A} \cdot \mathbf{A}^+ \cdot \mathbf{F_B^d} = \mathbf{F_B^d}, \ \forall r_m$$

(for some nice properties of A)



6. Actuation layer robustness

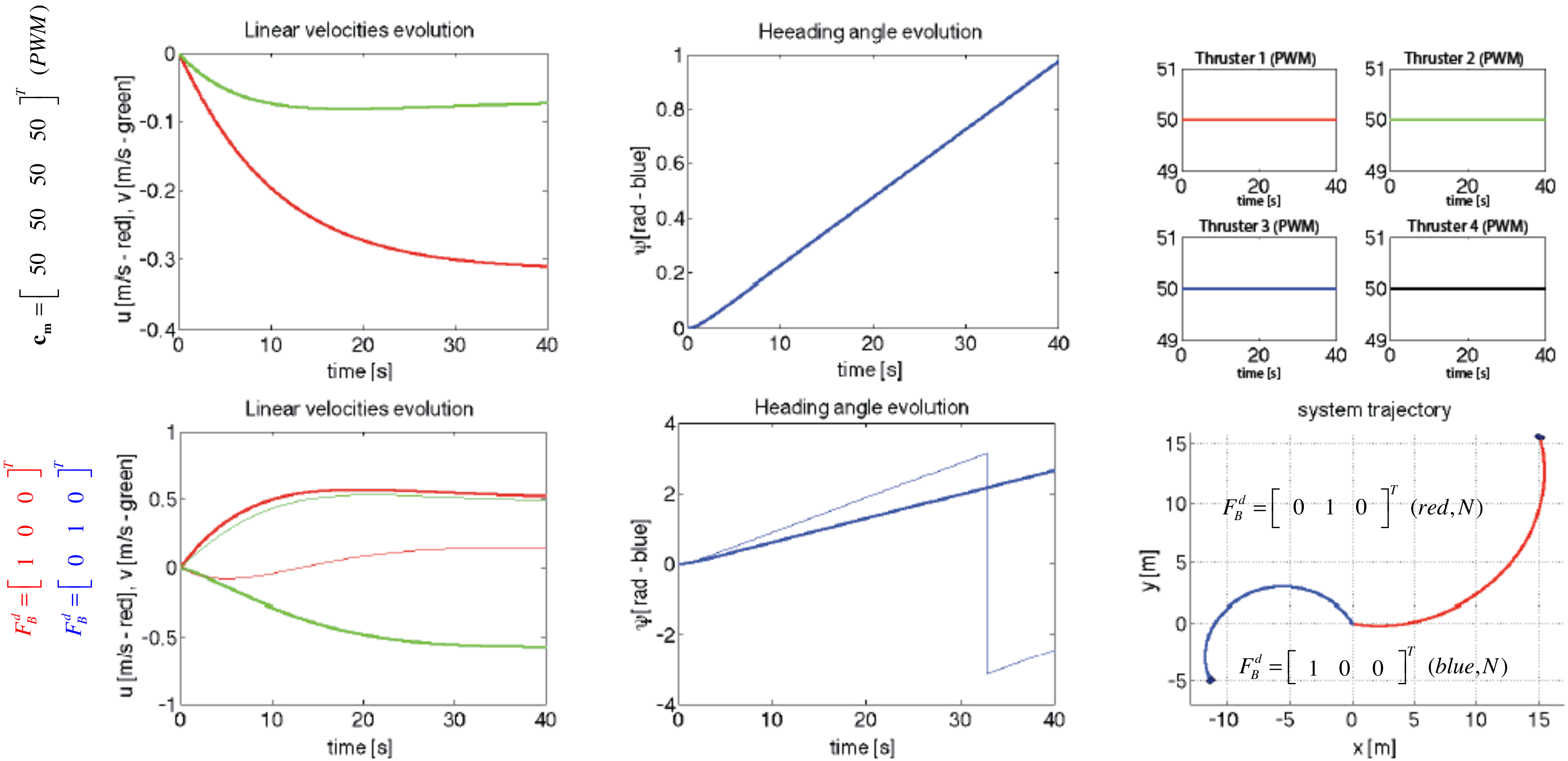


6.a Consider the motors' characteristic uncertainty and disparity

$$\mathbf{F_m} = \Omega(\mathbf{c_m}) \qquad \mathbf{c_m} = \hat{\Omega}^{-1}(\mathbf{F_m})$$

$$\mathbf{F}_B = \mathbf{A}\cdot\Omega\left(\hat{\Omega}^{-1}\left(\mathbf{A}^{+}\cdot\mathbf{F_B^d} + \mathbf{M_m}\cdot r_m\right)\right) \equiv \mathbf{A}\cdot\Omega\cdot\hat{\Omega}^{-1}\cdot\left(\mathbf{A}^{+}\cdot\mathbf{F_B^d} + \mathbf{M_m}\cdot r_m\right) \neq \mathbf{F_B^d}$$

➔DOF Coupling effect



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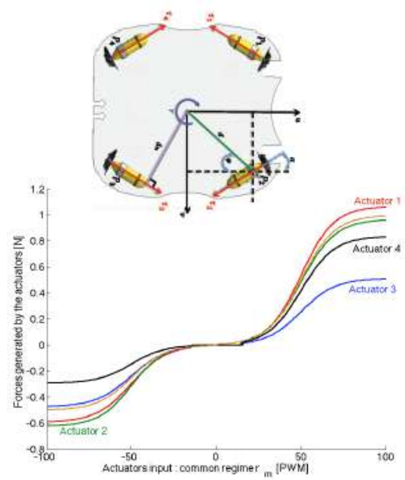

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➔DOF Coupling effect

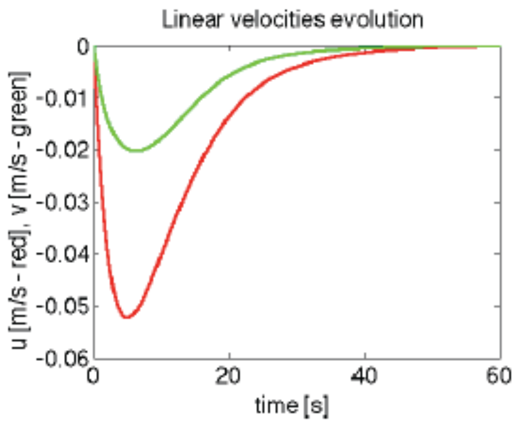
6.b Consider the following closed loop control

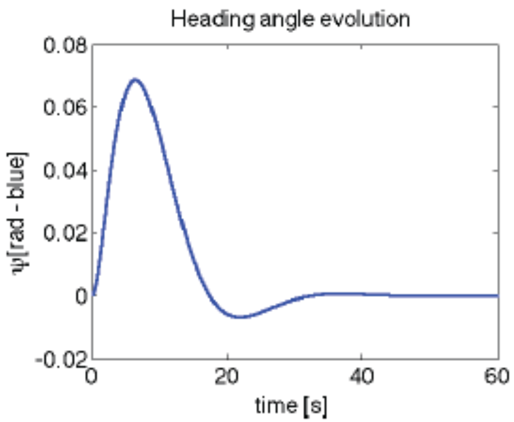
$$\mathbf{F_B^d} = \begin{bmatrix} -u - \int_0^t u\cdot \mathrm{d}t \\ -v - \int_0^t v\cdot \mathrm{d}t \\ -\psi - r - 0.1\cdot\int_0^t \psi\cdot \mathrm{d}t \end{bmatrix}, \begin{cases} \mathbf{c_m} = \hat{\boldsymbol{\Omega}}^{-1}\cdot\mathbf{A}^+\cdot\left(\mathbf{F_B^d} + \mathbf{M_m}\cdot r_m\right) \\ r_m = \Omega\cdot c_0 \end{cases}$$

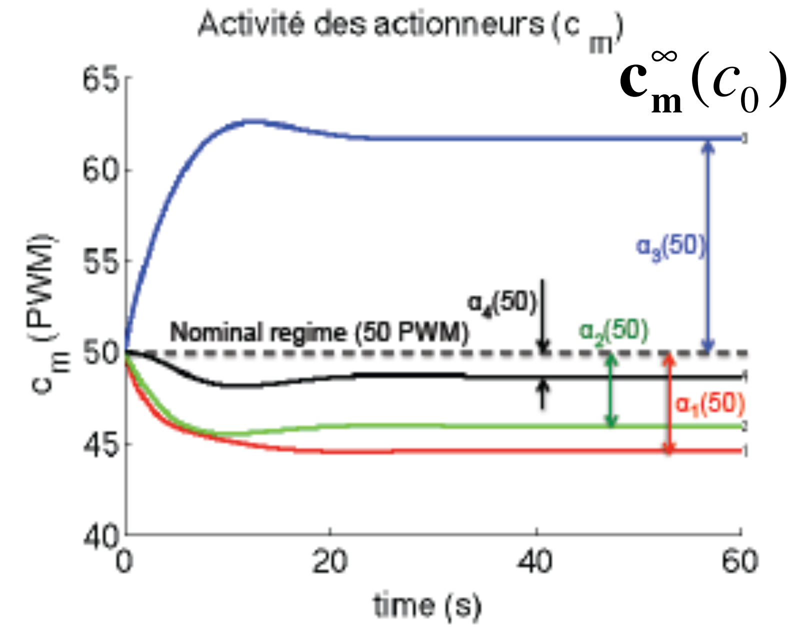
$$\mathbf{F_B} = \mathbf{A}\cdot\Omega\cdot\mathbf{c_m^\infty}(c_0) = \mathbf{0}$$

$$\Rightarrow \mathbf{c_m^\infty}(c_0) \in \ker(\mathbf{A}\cdot\Omega)$$

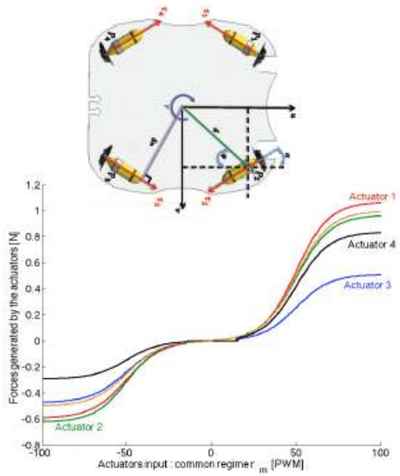
$$\Rightarrow \alpha_i(c_0) = \frac{c_{\mathbf{m},i}^\infty}{c_0}$$







6. Actuation layer robustness



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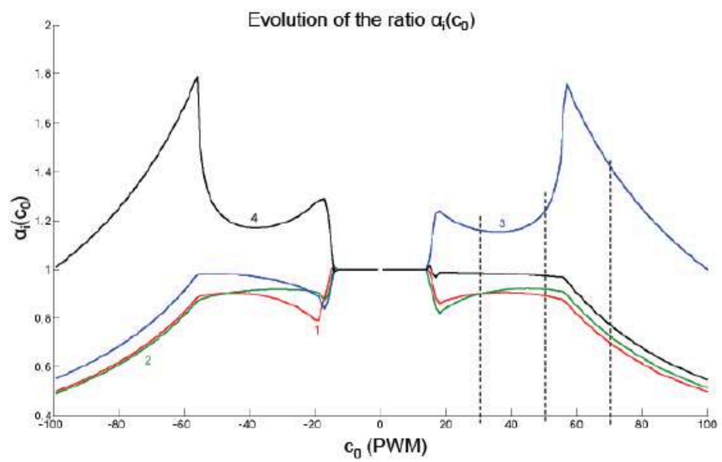
$$\Rightarrow \mathbf{c_m^\infty}(c_0) \in \ker(\mathbf{A}\cdot\Omega)$$

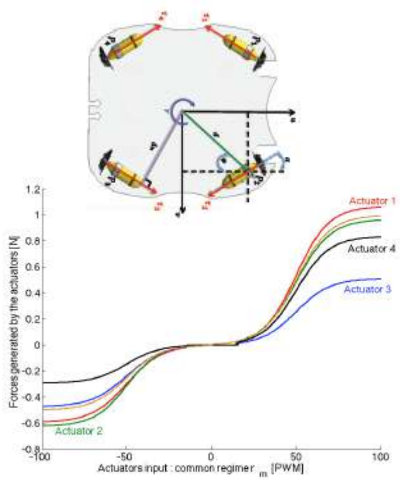
$$\Rightarrow \alpha_i(c_0) = \frac{c_{\mathbf{m},i}^\infty}{c_0}$$

$$\Rightarrow \mathbf{Q}(\mathbf{c_m}) = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$

6.c Iterate for $c_{m,\min} < c_m < c_{m,\max}$



6. Actuation layer robustness



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$$\mathbf{F_B} = \mathbf{A}\cdot\Omega\cdot\mathbf{c_m^\infty}(c_0) = \mathbf{0}$$

$$\Rightarrow \mathbf{c_m^\infty}(c_0) \in \ker(\mathbf{A}\cdot\Omega)$$

$$\Rightarrow \alpha_i(c_0) = \frac{c_{\mathbf{m},i}^\infty}{c_0}$$

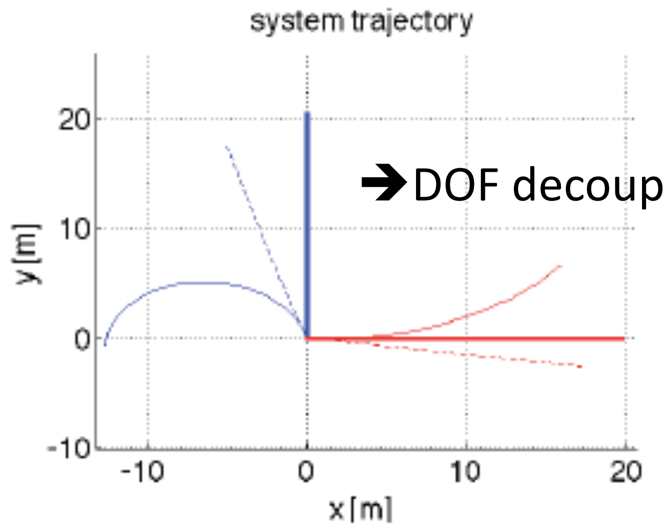
$$\Rightarrow \mathbf{Q}(\mathbf{c_m}) = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$

6.c Iterate for $c_{m,\min} < c_m < c_{m,\max}$

6.d Implement the following open loop control

$$\mathbf{c_m} = \mathbf{Q}\left(\cdot\hat{\boldsymbol{\Omega}}^{\mathbf{-1}}\cdot\mathbf{A}^+\cdot\left(\mathbf{F_B^d} + \mathbf{M_m}\cdot r_m\right)\right)$$

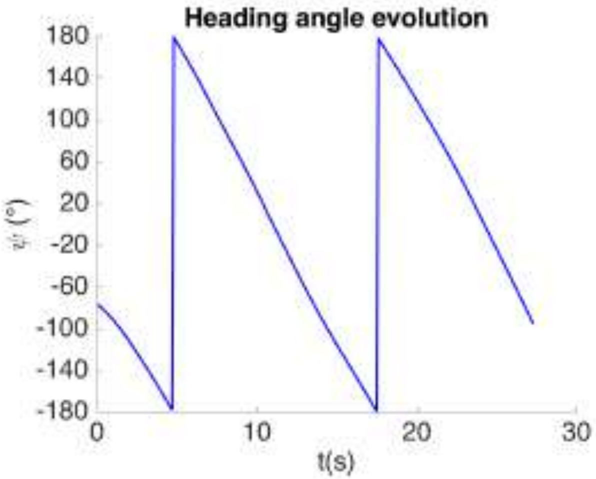
$$\mathbf{F_B} \equiv \mathbf{A}\cdot\boldsymbol{\Omega}\cdot\mathbf{Q}\cdot\hat{\boldsymbol{\Omega}}^{\mathbf{-1}}\cdot\begin{bmatrix} \mathbf{A}^+ & M_m \end{bmatrix}\cdot\begin{bmatrix} \mathbf{F_B^d} \\ r_m \end{bmatrix} = k_Q\cdot\mathbf{F_B^d}$$

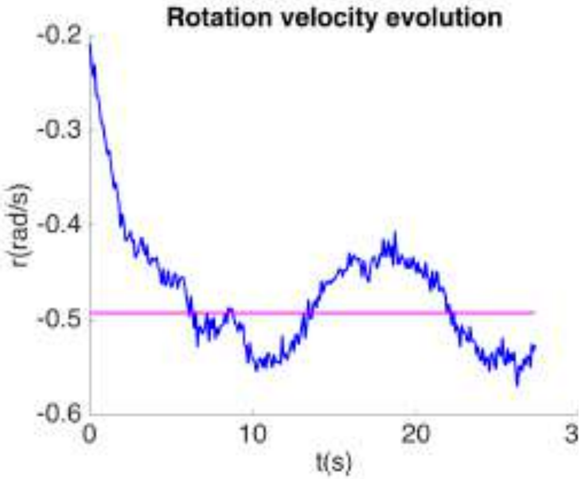


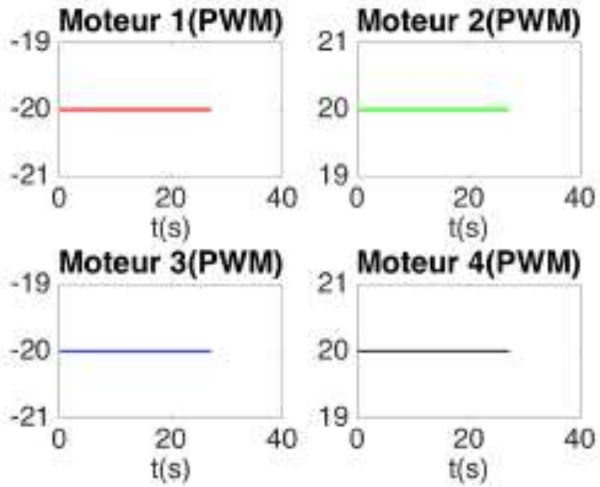
➔DOF decoupled

6. Actuation layer robustness

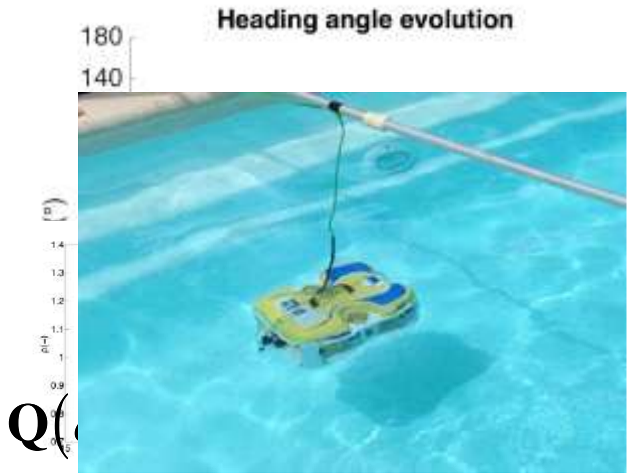
$$\mathbf{c_m} = \hat{\mathbf{\Omega}}^{-1} \cdot \mathbf{A}^{+} \cdot \left(\mathbf{0} + \mathbf{M_m} \cdot r_m\right)$$

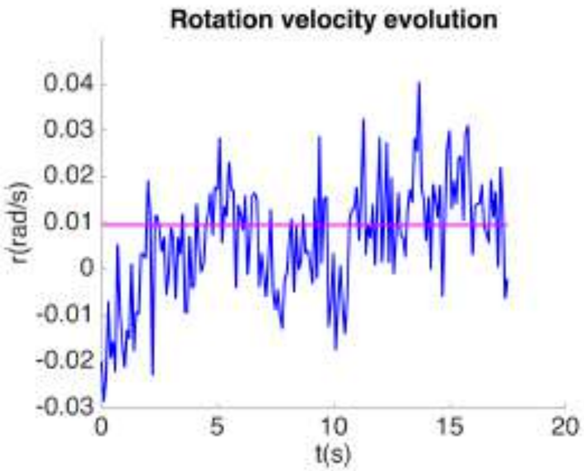


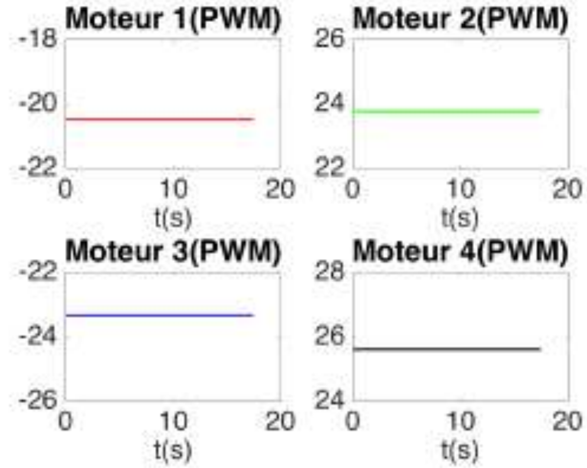




$$\mathbf{c_m} = \mathbf{Q}\left(\cdot \hat{\mathbf{\Omega}}^{-1} \cdot \mathbf{A}^{+} \cdot \left(\mathbf{0} + \mathbf{M_m} \cdot r_m\right)\right)$$







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